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NATIONAL SENIOR CERTIFICATE

BUFFALO CITY METRO DISTRICT

GRADE 12

MATHEMATICS GAME CHANGER

2025

THIS BOOKLET CONSISTS OF 64 PAGES.

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THEOREM PROOFS

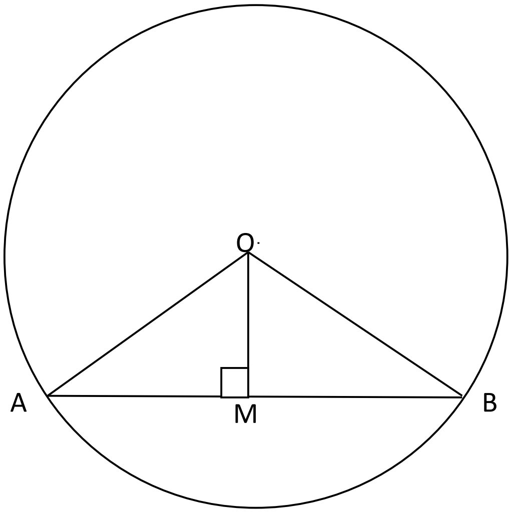
1A] IF A LINE IS DRAWN FROM THE CENTRE OF A CIRCLE PERPENDICULAR TO A CHORD, THEN IT BISECTS THE CHORD.

Given: Circle with centre O and chord AB with OM perpendicular to AB**.**

RTP: AM = AB

Constr: Draw OA and OB.

STEP 1: Construction



STEP 2: Prove congruency

Proof: In ΔOAM and ΔOBM

1] $O\hat{M}A = O\hat{M}B = 90°$ (given)
2] OA = OB (radii)
3] OM is common
∴ ΔOAM ≡ ΔOBM (90°; hyp; s)
∴ AM = BM

STEP 3: Deduce equal sides

DEDUCTION:

The perpendicular bisector of a chord passes through the centre of the circle
[For any point O, on the perpendicular bisector of AB, OA = OB as ΔOAM ≡ ΔOBM (s,a,s) therefore in particular the centre must lie on the perpendicular bisector to provide for OA and OB being equal radii]

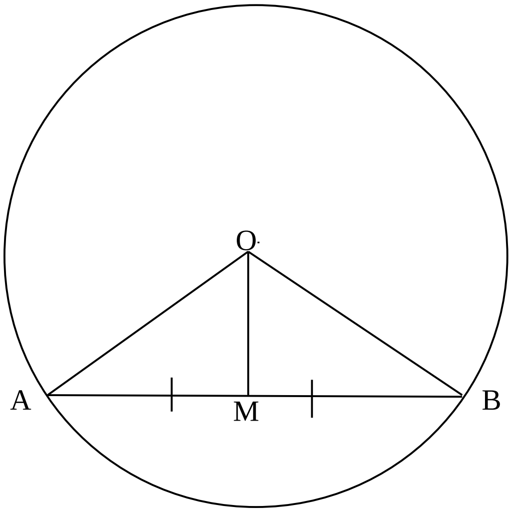
1B] IF A LINE IS DRAWN FROM THE CENTRE OF A CIRCLE TO THE MIDPOINT OF A CHORD, THEN IT IS PERPENDICULAR TO THE CHORD.

Given: Circle with centre O and chord AB with M the midpoint of AB**.**

RTP: $OM \perp AB$

Constr: Draw OA and OB.

STEP 1: Construction



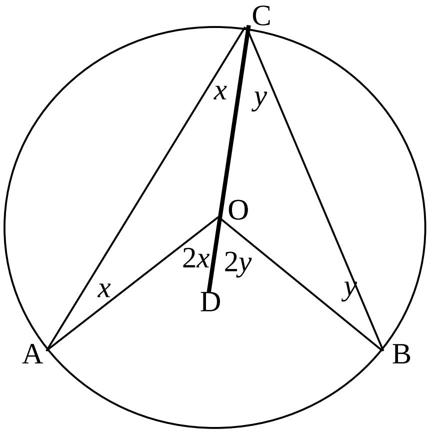
STEP 2: Prove congruency

Proof: In ΔOAM and ΔOBM

1] AM = BM (given)
2] OA = OB (radii)
3] OM is common
$\therefore$ $\Delta OAM \equiv \Delta OBM$ (s; s; s)
$\therefore$ $A\hat{M}O = B\hat{M}O = 90°$ (AMB is a straight line)

STEP 3: Deduce equal right angles

2] IF A CHORD (OR ARC) SUBTENDS ANGLES AT THE CENTRE AND ON A CIRCLE, THEN THE ANGLE AT THE CENTRE IS DOUBLE THE ANGLE ON THE CIRCLE



Given: Circle, centre O, with arc AB subtending AÔB at the centre and $A\hat{C}B$ on the circle.

Constr: Join CO and extend to D

STEP 1: Construction

R.T.P: $A\hat{O}B = 2A\hat{C}B$

Proof: Let $A\hat{C}D = x$

$\therefore C\hat{A}O = x$ (angles opp. equal sides)

STEP 2: Show AÔD = 2AĈD

$\therefore A\hat{O}D = 2x$ (ext. angle of triangle = sum int. opp. angles)

Similarly, if $B\hat{C}D = y$, then $B\hat{O}D = 2y$

STEP 3: Show BÔD = 2BĈD

$$\begin{aligned}\therefore A\hat{O}B &= 2x + 2y \\ &= 2(x+y) \\ &= 2A\hat{C}B\end{aligned}$$

STEP 4: Common factor and conclusion

DEDUCTIONS:

- Angles subtended by a chord / arc in the same segment are equal

[they are all half the angle subtended by the chord / arc at the centre]

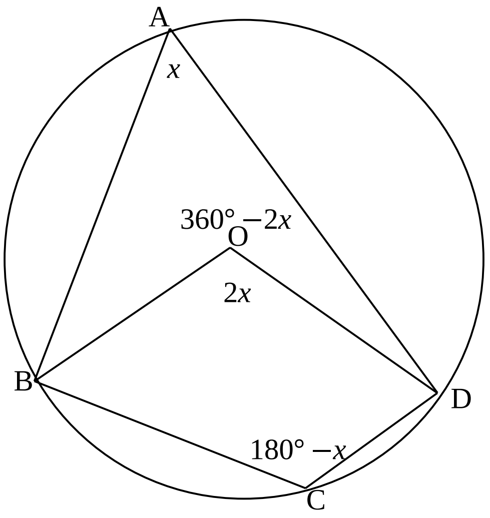
- The angle in a semi-circle is 90°

[it is subtended by a diameter and so is half of 180°]

- Equal chords subtend equal angles at centre and on circle

3] IF A QUAD. IS CYCLIC, THEN ITS OPPOSITE ANGLES ARE SUPPLEMENTARY (ADD UP TO 180º).

Given: Cyclic quad. ABCD.



R.T.P.: $\hat{A}+\hat{C}=180^\circ$ and $\hat{B}+\hat{D}=180^\circ$

Constr: Draw in BO and DO

STEP 1: Construction

Proof: Let $\hat{A}=x$

then obtuse $B\hat{O}D = 2x$ (angle at centre = twice angle on circle)

$\therefore$ reflex $B\hat{O}D = 360^\circ - 2x$ (revolution)

STEP 2: Express angles at O in terms of *x*

$\therefore$ $\hat{C} = 180^\circ - x$ (angle on circle = half angle at centre)

$\therefore$ $\hat{A}+\hat{C} = x + (180^\circ - x)$

$= 180^\circ$

STEP 3: Express angle at C in terms of *x* and add to opp. angle

and $\hat{B}+\hat{D}=180^\circ$ (4 angles of quad. add up to 360º)

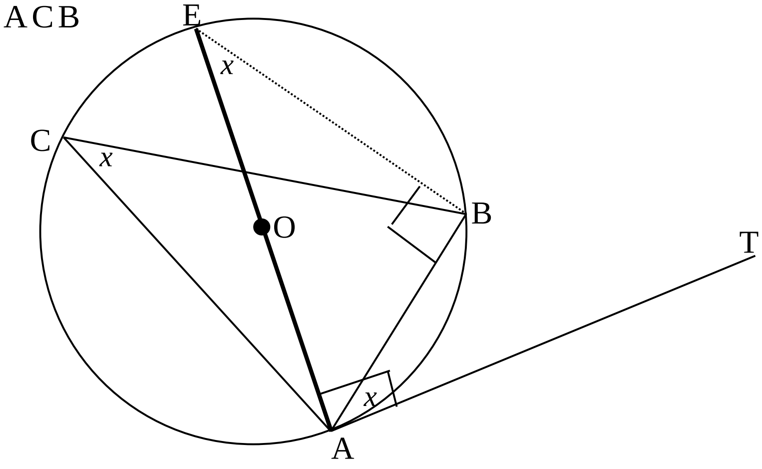
DEDUCTIONS

- The exterior angle of a cyclic quadrilateral equals the interior opposite angle
- Converses: If the exterior angle of a quadrilateral equals the interior opposite angle, then the quadrilateral is cyclic
- If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic

4] IF A TANGENT AND CHORD MEET ON A CIRCLE, THEN THE ANGLE BETWEEN THE TANGENT AND CHORD EQUALS THE ANGLE IN THE ALTERNATE SEGMENT.

Given: Circle with centre O, chord AB, tangent AT and $A\hat{C}B$ in the alternate segment.

R.T.P: $T\hat{A}B = A\hat{C}B$



Constr: Draw in **diameter** AE and join EB.

STEP 1: Construction of **diameter**

Proof: Let $T\hat{A}B = x$

$T\hat{A}E = 90°$ (angle between diameter and tangent)

and $A\hat{B}E = 90°$ (angle in semi-circle)

STEP 2: Deduce two right angles

$\therefore E\hat{A}B = 90° - x$

$\therefore \hat{E} = x$ (angles of triangle sum to 180°**)**

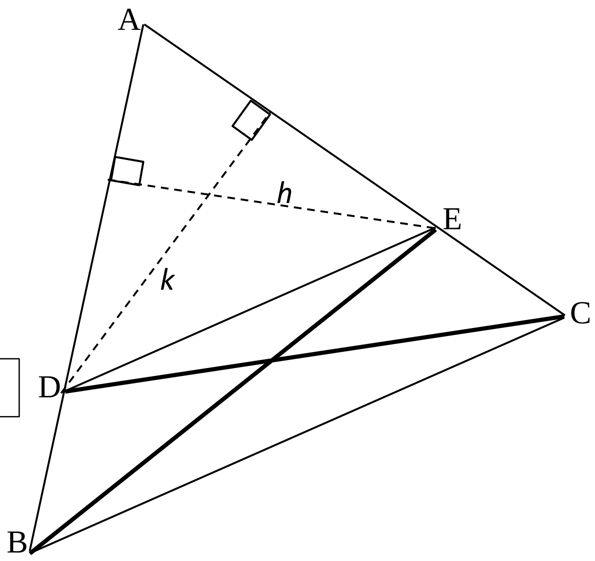
$\therefore \hat{C} = x$ (same segment)

STEP 3: Deduce $E\hat{A}B$ and equal angles at E and C

DEDUCTION

- Tangents drawn from a common point outside a circle are equal in length

5] IF A LINE IS DRAWN PARALLEL TO ONE SIDE OF A TRIANGLE, THEN IT DIVIDES THE OTHER 2 SIDES IN PROPORTION.



Given: ΔABC with DE∥BC.

R.T.P. : $\frac{AD}{DB}=\frac{AE}{EC}$

Construction: Draw DC and BE.

STEP 1: Construction

Proof:

$$\frac{\text{Area}\Delta\text{EDA}}{\text{Area}\Delta\text{EBD}}=\frac{\frac{1}{2}\text{AD}\times h}{\frac{1}{2}\text{DB}\times h}=\frac{\text{AD}}{\text{DB}}$$

STEP 2: Work from E as vertex and AD & DB as bases

$$\frac{\text{Area}\Delta\text{DEA}}{\text{Area}\Delta\text{DEC}}=\frac{\frac{1}{2}\text{AE}\times k}{\frac{1}{2}\text{EC}\times k}=\frac{\text{AE}}{\text{EC}}$$

STEP 3: Work from D as vertex and AE & EC as bases

but, Area ΔEDA = Area ΔEDA and

$\text{Area}\Delta\text{EBD}=\text{Area}\Delta\text{DEC}$ (*same base* and between same parallel lines)

STEP 4: Prove areas equal

$$\therefore\frac{\text{Area}\Delta\text{EDA}}{\text{Area}\Delta\text{EBD}}=\frac{\text{Area}\Delta\text{EDA}}{\text{Area}\Delta\text{ECD}}$$

$$\therefore\frac{\text{AD}}{\text{DB}}=\frac{\text{AE}}{\text{EC}}$$

STEP 5: Deduce ratios equal

- To use the theorem, you use the fact that the lines are parallel and conclude the sides are divided in proportion. The reason must include both the name of the theorem and the parallel lines. For example

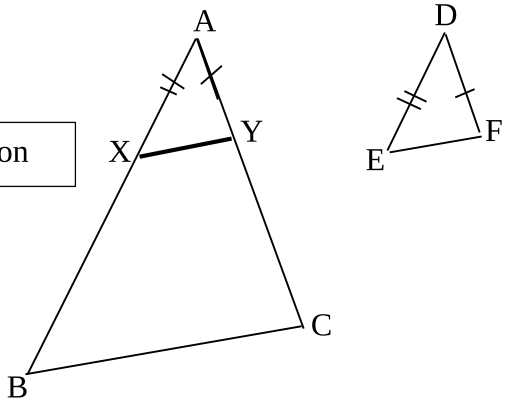
  $\therefore\frac{\text{AD}}{\text{DB}}=\frac{\text{AE}}{\text{EC}}$ (Proportional Intercepts; DE∥BC)

SIMILAR TRIANGLES

For triangles to be similar, they must be equiangular and their corresponding sides must be in proportion.

The theorem below shows that if the triangles are equiangular, then their sides are also in proportion, so proving equiangular is sufficient.

6] IF TRIANGLES ARE EQUIANGULAR, THEN THEIR SIDES ARE IN PROPORTION AND THEY ARE SIMILAR



Given: ΔABC and ΔDEF with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$

R.T.P. : $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

STEP 1: Construction

Construction: On ΔABC cut off AX = DE and AY = DF. Join XY.

Proof:

In ΔAXY and ΔDEF

1. AX = DE (const.)
2. $\hat{A} = \hat{D}$ (given)
3. AY = DF

∴ ΔAXY ≡ ΔDEF (s ; < ; s)

STEP 2: Prove triangles congruent

∴$A\hat{X}Y = \hat{E}$ (congruent triangles)

$= \hat{B}$ (given)

STEP 3: Prove that corresponding angles are equal

∴XY || BC (corresp. <'s equal)

STEP 4: Deduce parallel lines

$$\therefore \frac{AB}{AX} = \frac{AC}{AY} \quad \text{(proportional intercepts theorem)}$$

STEP 5: Apply proportional intercepts theorem

$$\therefore \frac{AB}{DE} = \frac{AC}{DF} \quad (AX = DE\,;\ AY = DF)$$

The rest is for completeness, but rarely asked.

Similarly, by cutting off lengths equal to ED and EF from B, it can be shown that $\frac{AB}{DE} = \frac{BC}{EF}$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

∴ ΔABC ||| ΔDEF [equiangular and sides in prop.]

MATHGYM 21/8

1. Solve for *x*:

1,1 $5x^2 - 10x = 0$ (2)

1.2 $5x^2 - 10x > 0$ (2)

1.3 $5x^2 - 10x + 4 = 0$ [answer rounded to 2 decimal digits] (3)

2. Calculate the values of p for which $5x^2 - 10x + p = 0$ has no real roots. (3)

[10]

1.1	$5x^2 - 10x = 0$ $\therefore x^2 - 2x = 0$ $\therefore x(x-2) = 0$ $\therefore x = 0 \;\; or \;\; x = 2$	✓factors ✓answers	(2)
1.2	$x(x-2) > 0$ $\therefore x < 0 \;\; or \;\; x > 2$	✓✓answer	(2)
1.3	$5x^2 - 10x + 4 = 0$ $\therefore x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(5)(4)}}{2(5)}$ $\therefore x = 1{,}45$ or $x = 0{,}55$	✓substitution ✓1,45 ✓0,55	(3)
2	$5x^2 - 10x + p = 0$ roots not real $\therefore \Delta < 0$ $\therefore 100 - 20p < 0$ $\therefore p > 5$	✓$\Delta < 0$ ✓$100 - 20p$ ✓answer	(3) **[10]**

MATHGYM 22/8

1 Determine $\frac{dy}{dx}$ if:

1.1 $y = 2x^{20} + 20\sqrt{x}$ (2)

1.2 $(x+1)y = x^3 + 1$ if $x \neq -1$ (3)

2. Determine the derivative of $f(x) = x^2 - x$ from first principles (5)

[10]

solutions

1.1	$y = 2x^{20} + 20\sqrt{x}$ $\therefore y = 2x^{20} + 20x^{\frac{1}{2}}$ Notice l.h.s. is still y = . . . as in this 1st step we are only writing r.h.s. as row of powers of x Only once we differentiate does l.h.s. become $\frac{dy}{dx}$ $\therefore \frac{dy}{dx} = 40x^{19} + 10x^{-\frac{1}{2}}$	✓$40x^{19}$ ✓$10x^{-\frac{1}{2}}$	(2)
1.2	$(x+1)y = x^3 + 1$ $\therefore (x+1)y = (x+1)(x^2 - x + 1)$ $\therefore y = x^2 - x + 1$ $\therefore \frac{dy}{dx} = 2x - 1$	✓factorising “sum of cubes” ✓division ✓answer	(3)
2	$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$ $= \lim_{h\to 0}\frac{(x+h)^2 - (x+h) - [x^2 - x]}{h}$ $= \lim_{h\to 0}\frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h}$ $= \lim_{h\to 0}\frac{2xh + h^2 - h}{h}$ $= \lim_{h\to 0}(2x + h - 1)$ $= 2x - 1$	MAKE SURE NOTATION IS EXACTLY LIKE THIS ✓substitution ✓products ✓simplification ✓division by *h* ✓limit	(5)

MATHGYM 25/8

Players in a rugby team are numbered from 1 to 15 according to their positions. A coach collected data from a sample of 10 players in order to research the relationship between player numbers and the number of tries the players scored in a series of matches. The data is shown in the table below.

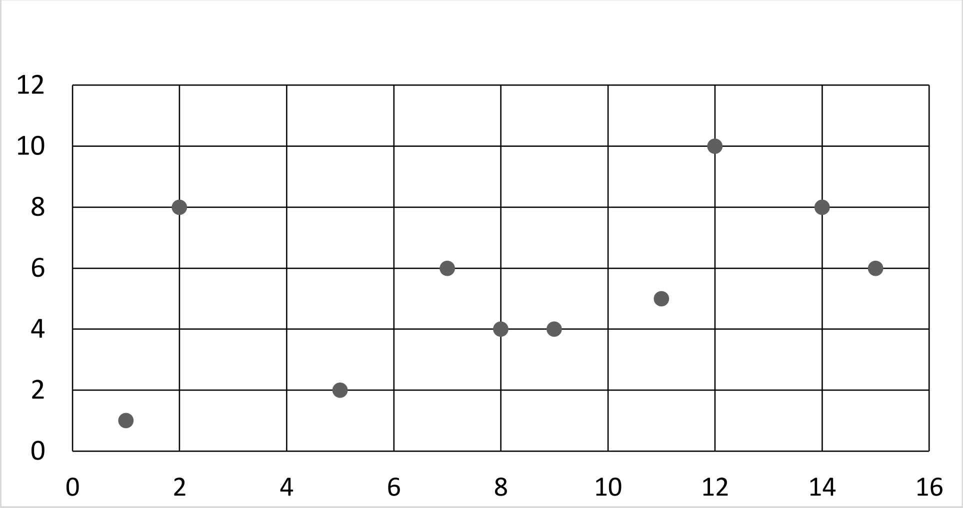
Player number	1	2	5	7	8	9	11	12	14	15
number of tries	1	8	2	6	4	4	5	10	8	6

1. Determine the equation of the least squares regression line. (3)

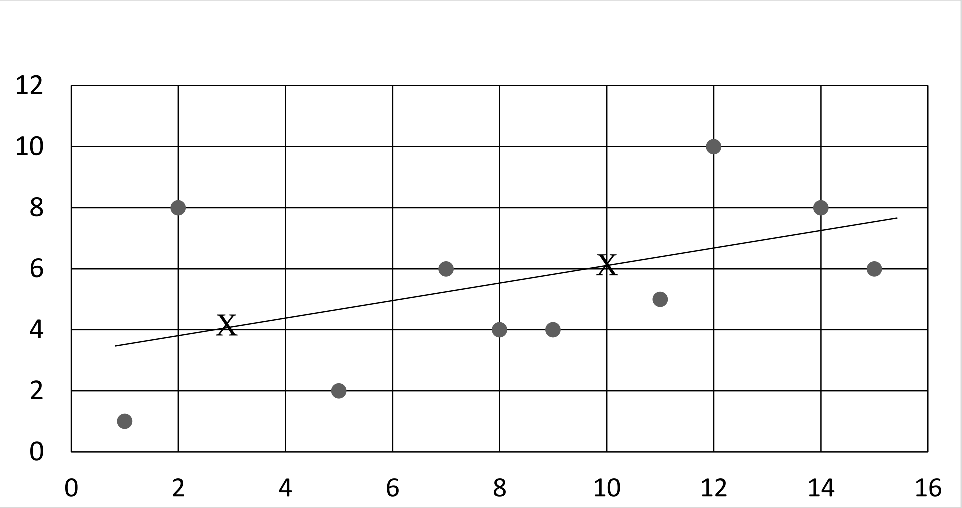
2. Which of the following descriptions best fit the correlation between player number and the number of tries scored? Motivate your answer by referring to the correlation coefficient. (2)
   A. no correlation B. weak positive C. moderate positive D. weak negative

3. Use the equation of the least squares regression line to predict the number of tries scored by players 3 and 10. Hence, draw the least squares regression line on the scatterplot below. (3)

4. Which player can be identified as the greatest outlier? Give a reason for your answer. (2)
   Bonus for rugby fans: What can you deduce about the tactic used by the team from attacking line-outs?



SOLUTIONS:



1	$\hat{y} = 2{,}92 + 0{,}30\,x$	✓2,92 ✓0,3 ✓equation	(3)
2	C as $r = 0{,}50$	✓C ✓0,5	(2)
3	$\hat{y} = 2{,}92 + 0{,}30\,(3) = 3$ $\hat{y} = 2{,}92 + 0{,}30\,(10) = 6$	✓3 ✓6 ✓correct line and domain.	(3)
4	player 2 as (2 ; 8) is the point which is furthest from the line.	✓2 ✓reason	(2) [10]
	rolling maul allowing hooker to score so many tries	bonus	

MATHGYM 26/8

The general term of an Arithmetic Sequence is $T_n = 4n - 7$

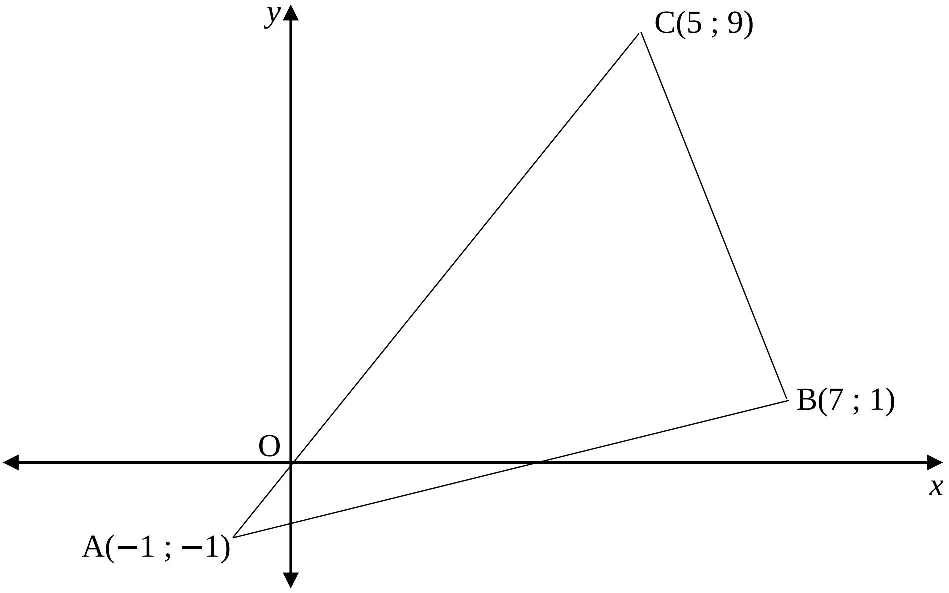
1. Determine the value of T_1. (2)
2. Determine the sum of the first 2 terms, S_2 (2)
3. Show that the sum of the first *n* terms is given by $S_n = 2n^2 - 5n$. (2)
4. Show that S_1; S_2; S_3 and S_4 form a quadratic pattern. (4)

SOLUTIONS

1	$T_1 = 4(1) - 7 = -3$	✓substitution ✓value	(2)
2	$T_2 = 1$ $S_2 = -3 + 1 = -2$	✓T_2 ✓sum	(2)
3	$a = -3$ $d = 4$ $S_n = \frac{n}{2}[2(-3) + (n-1)4] = \frac{n}{2}[4n - 10]$ $= 2n^2 - 5n$	✓d ✓substitution	(2)
4	$S_1 = -3$ $S_2 = -2$ $S_3 = 3$ $S_4 = 12$ First differences are: 1; 5 *and* 9 Second difference is constant 4 which shows that pattern is quadratic. Note: the fact that the general term is a quadratic expression also shows this.	✓S_3 ✓S_4 ✓first differences ✓constant second difference	(4)

MATHGYM 27/8

In the diagram A(−1 ; −1) , B(7 ; 1) and C(5 ; 9) are the vertices of ΔABC.



1. Calculate the gradients of AB and BC	(2)
2. Prove that $\widehat{B}$ is 90°	(1)
3. Why is it possible to conclude that AC is the diameter of circle ABC?	(1)
4. Determine the equation of the circle through A, B and C	(4)
5. Determine the equation of line BC	(2)
	[10]

solutions

1. $m_{AB} = \frac{1-(-1)}{7-(-1)} = \frac{2}{8} = \frac{1}{4}$ $m_{BC} = \frac{9-1}{5-7} = -4$ ✓ m_{AB} ✓ m_{BC} (2)

2. $m_{AB} \times m_{BC} = \frac{1}{4} \times (-4) = -1$ ✓product and conclusion

$\therefore \hat{B}$ is 90° (1)

3. AC subtends a right-angle at B ✓reason (1)

4. Centre is midpoint of diameter, so M(2 ; 4) ✓✓midpoint

radius = MC [or MA or MB]

$= \sqrt{(5-2)^2 + (9-4)^2} = \sqrt{34}$ ✓length of radius

equation is $(x-2)^2 + (y-4)^2 = 34$ ✓equation (4)

5. $m_{BC} = -4$ B(7 ; 1)

$y - 1 = -4(x - 7)$ ✓substitution

$y = -4x + 29$ ✓answer (2)

MATHGYM 28/8

A rugby coach selects a reserve bench comprising of 5 forwards and 3 backs.
These players are required to sit in a row next to the field.

1. In how many different ways can they be seated if there are no restrictions? (1)

2. In how many different ways can they be seated if the backs must all sit next to one another? (2)

3. In how many different ways can they be seated if no backs may sit next to each other? (3)

4. What is the probability that all the backs sit next to one another and all the forwards sit next to one another? (4)

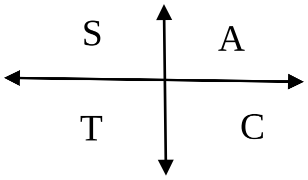
[10]

SOLUTIONs

1	the first player has 8 options, the second 7 and so on. Therefore, the answer is 8! = 40 320	✓8! or 40 320	(1)
2	The backs form an **entity** containing 3 individuals. This entity plus the 5 forwards make 6 units to be arranged. Answer: 6! × 3! = 4 320	✓6! ✓3!	(2)
3	Backs must sit in **gaps** separated by forwards. _F_F_F_F_F_ . There are 6 such gaps. So the 1st back has 6 options, the 2nd has 5 and the 3rd has 4. There are 5! ways in which the 5 forwards can be distributed. Answer: 6 × 5 × 4 × 5! = 14 400	✓6 × 5 × 4 ✓5! ✓answer	(3)
4	the backs form an entity and the forwards form an entity. The 2 entities must be arranged. Number of arrangements: 2! × 3! × 5! = 1440 Probability $= \frac{1440}{40320} = \frac{1}{28}$ or 0,04	✓2 ✓3! × 5! ✓division by 40 320 ✓answer	(4) [10]

MATHGYM 29/8

The diagram indicates which trigonometric ratio is positive in each quadrant and can be used to assist in answering the questions.



1. Express each ratio in terms of an acute angle and simplify without using a calculator:

$$\sin(-130^\circ) . \sin 310^\circ + \cos 410^\circ . \cos(-50^\circ) \qquad (6)$$

2. Simplify to a single trigonometric ratio:

$$\frac{\sin(180^\circ - A) + \sin A}{\cos(720^\circ + A)}$$

(4)

Solutions

1	$\sin(-130°)\,.\sin 310° + \cos 410°\,.\cos(-50°)$ 3rd quad so negative and 50° from closest x-axis 4th quad so negative and 50° from closest x-axis 1st quad so positive and 50° from closest x-axis 4th quad so positive and 50° from closest x-axis $= -\sin 50°\ .(-\sin 50°) + \cos 50°\ .\cos 50°$ $= \sin^2 50° + \cos^2 50°$ $= 1$	✓$-\sin 50°$✓$-\sin 50°$ ✓$\cos 50°$ ✓$\cos 50°$ ✓$\sin^2 50° + \cos^2 50°$ ✓ identity: $\sin^2 50° + \cos^2 50° = 1$	(6)
2	2nd quad so positive $\dfrac{\sin(180° - A) + \sin A}{\cos(720° + A)}$ 1st quad so positive $= \dfrac{\sin A + \sin A}{\cos A}$ $= \dfrac{2\sin A}{\cos A}$ $= 2\tan A$	✓$\sin A$ ✓$\cos A$ ✓$2\sin A$ ✓identity: $\dfrac{\sin A}{\cos A} = \tan A$	(4)

MATHGYM 1/9

The following information is given about a quadratic number pattern.

- $T_n = n^2 + bn + c$
- $T_1 = 22$
- $T_2 = 20$

Determine:

1. The value of the first term of the sequence of first differences. (1)

2. The value of the second difference (1)

3. The values of b and c. (4)

4. The value of the 25th term of the quadratic number pattern. (2)

5. The sum of the first 24 terms of the sequence of first differences. (2)

SOLUTIONS

1	20 – 22 = -2	✓-2	(1)
2	2^{nd} difference = 2 × the coefficient of n^2 = 2	✓2	(1)
3	$T_1 = 22$ $\rightarrow\ 1^2 + b(1) + c = 22\ \ \therefore b + c = 21\ .\ .\ .(1)$ $T_2 = 2$ $\rightarrow 2^2 + b(2) + c = 20\quad \therefore 2b + c$ $= 16\ .\ .\ .(2)$ (2) – (1) : $b = -5$ subst. in (1): $-5 + c = 21\ \ \therefore c = 26$ OR $T_2 - T_1 = -2\ \ \therefore 3 + b = -2\ \ \therefore b = -5$ and $b + c = 21\quad \therefore -5 + c = 21\ \ \therefore c = 26$	✓equation ✓equation ✓b value ✓c value ✓equation ✓b value ✓equation ✓c value	(4)
4	$T_n = n^2 - 5n + 26$ $T_{25} = 25^2 - 5(25) + 26 = 526$	✓substitution ✓answer	(2)
4	sum of first 24 terms of first difference series = $T_{25} - T_1$ of Quadratic pattern = 526 – 22 = 504 OR first difference sequence is arithmetic with first term a = -2 and common difference d = 2 note that d = 2^{nd} diff of the Quadratic pattern so sum of first 24 term is $\frac{24}{2}[2(-2) + (24 - 1)(2)]$ $= 504$	✓method ✓answer ✓substitution in sum formula ✓answer	(2)

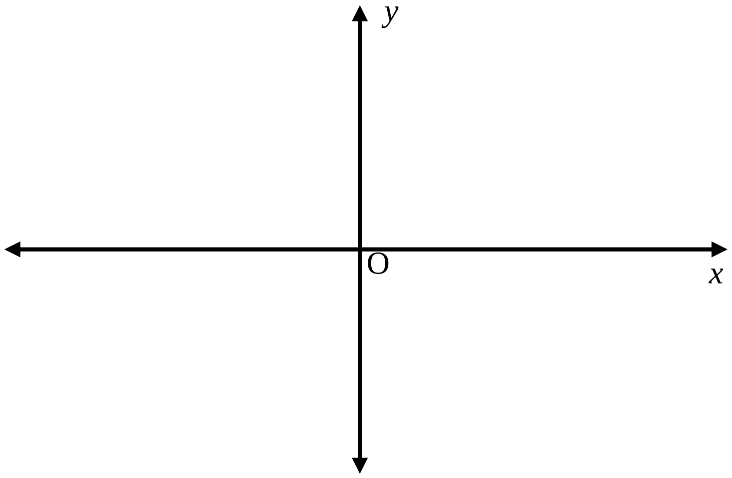
MATHGYM 2/9

Given: $f(x)=\frac{4}{x-2}+1$

1. Calculate the x-intercept of f. (2)

2. Write down the coordinates of the point where the axes of symmetry of f intersect. (1)

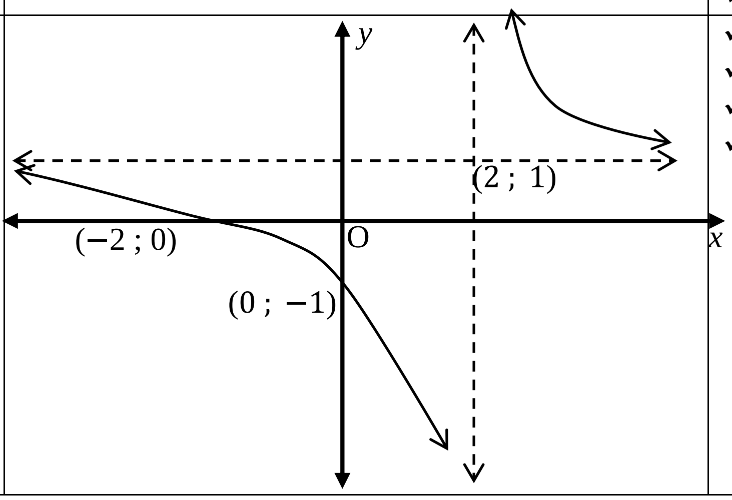
3. Sketch the graph of f on the set of axes below. Indicate all asymptotes and the coordinates of intercepts with the axes. (4)



4. Determine the equation of the axis of symmetry of f, that has negative gradient. (2)

5. Write down the equation of the vertical asymptote of $y=f(x+5)$. (1)

Solutions

1	$0=\frac{4}{x-2}+1$ $0=4+x-2$ $x=-2$	✓$y=0$ ✓$x=-2$	(2)
2	(2 ; 1)	Axes of symmetry intersect at the same point as asymptotes ✓answer	(1)
3		✓asymptotes ✓shape in correct quadrants ✓x-intercept ✓y-intercept	(4)

4	$y = -x + c$ Subst. (2 ; 1): $1 = -2 + c \quad \therefore c = 3$ $\therefore y = -x + 3$ OR $y = -(x - 2) + 1$ $\therefore y = -x + 3$	✓$m = -1$ and substitution of (2 ; 1) ✓answer OR ✓$m = -1$ and translation ✓answer	(2)
5	translation is 5 units left, so $x = -3$	✓equation	(1) **[10]**

GAME PLAN FOR MATHEMATICS EXAMS

1. THE NEED FOR CONSISTENCY OF ACTION: You perform best in a game if you manage your emotions. To win a final, you need consistency – you must do what you are used to doing so that tenseness or nervousness or fatigue do not inhibit giving your best. To help you achieve this state of readiness to do maths, doing a daily MATHGYM exercise is recommended. It should only take 12 minutes and will get you used to doing unseen, fresh problems as a matter of routine. The exam will then simply become a collection of such exercises. Examples of MATHGYM exercises with dates from 21 August to 2 September are attached..

2. THE NEED TO GO THROUGH YOUR SCHOOL NOTES: An analysis of your performance in earlier papers would indicate the need to go through your school notes BEFORE using other sources. The notes should cover knowledge needed for almost all questions types you could get. Many will already know their notes or have appropriate summaries. Then use the summaries of the various sections.
Make sure you know the derivations of the SUM formulae for Paper 1 and TRIANGLE and EXPANSION formulae for Paper 2. Also please LEARN the 7 THEOREMS.
Only when you have done this, practice old papers. The recent ones are best and are all on the ecexams,co,za website. Make notes of your errors during practice so that you learn from them.

Learning for Mathematics then becomes a cycle: Learning from Books and Notes; Learning from Practice; Learning from Mistakes. The cycle is repeated to fix mistakes.*

*THOSE WHOSE GOAL IS SIMPLY TO PASS SHOULD TAKE NOTE OF THE ADDITIONAL ADVICE IN THE ADDENDUM.

3. CHECK THE PAPER BREAKDOWN: It is in the exam guideline document.

4. THE DAY BEFORE PAPER 1:
   - Read the lessons from exam analysis postings and your own analysis of errors you specifically made in earlier exams so that you remind yourself not to repeat them.
   - Read the revision summaries of the past few weeks which also tried to eliminate common errors.
   - Read the instructions on a recent paper, to save yourself time in the exams – note in particular the instructions regarding rounding to 2 decimal places and showing working.
   - Go through the information sheet and ensure you know what each formula is for
   - Remind yourself of the formula and rules for Δ (nature of roots) and for effective vs nominal interest rate as well as of the rule for Independent and for Mutually exclusive events.
   - Make sure you are aware of the structure of the paper so you know what to look out for.
   - Get everything ready: a calculator, a blue / black ink pen to write with (NOTE THAT YOU MAY NOT WRITE IN PENCIL), a pencil for drawing graphs, an eraser, a ruler, YOUR ADMISSION LETTER AND ID.
   - Make sure arrangements are in place to get to school in good time.
   - Relax in the evening and sleep well.

5. THE 10 MINUTE READING TIME: As you read the paper, identify the easy questions. They should be most of Questions 1 (solving equations and inequalities) and 2 (number patterns); at least one of the graph questions; at least one part of the finance question; the first Calculus question (differentiating); parts of the cubic graph question; questions about independent or mutually exclusive events.

6. WHEN WRITING, start at Question 1 and try to maintain the order of the paper, but make sure that you answer all the questions you identified as “easy”. As there is a specific answer booklet, it should be possible to answer them in the spaces provided, before returning to complete more testing questions. Don’t get stuck on a question. Check that you don’t leave a question out by mistake.

7. NO sub-question is likely to count more that 6 marks {half a page or 7 minutes}. So if you are needing more time for one, you are probably wasting time. It is easy for this to happen when solving simultaneous equations if you have made a calculation error. If you don’t see it quickly, move on.

8. AFTERWARDS: Get some exercise to clear your head, and rest before tackling paper 2 revision.

9. WEEKEND: You have 2 days to do the activities indicated under 4. above. In addition

- check calculator is set on degrees
- check you know how to use statistics functions.
- LEARN THEOREMS
- learn derivations of triangle formulae (sine rule, etc.) and compound angle and double angle identities.
- make sure of all the little details in Statistics so that a very high score in this section is possible. [eg. ogive maps top of interval vs cumulative frequency, measures of dispersion]
- Go through revision notes on various sections

10. THE 10 MINUTE READING TIME for PAPER 2: Proceed as in 5. above. The easy questions are now likely to be Questions 1, 2 and 3. Note also the first trig question which usually refers to getting ratios from a diagram, the right-angled triangle calculations, the trig graphs, the first geometry question (calculations), a proof of similarity and the theorem.

11. WHEN WRITING PAPER 2, the booklet you answer in makes it easy to make sure you get the easy questions done first and do not run out of time. Be aware of time and don’t get stuck.
Make use of the diagrams [eg. if there is a ratio of areas question look for a common altitude or angle]
Check that you answer all questions.

12. MAKE SURE YOU ATTEND SCHOOL in the last week of term 3 and first week of term 4 so that your teacher can help you learn from your mistakes.

ADDENDUM: ADVICE TO SCORE ENOUGH TO PASS.

In each paper there are enough predictable, routine questions to obtain 50 marks. It is important to be fully prepared for these. Such questions are identified below and revision or summary sheets of such questions of a routine nature are attached.

PREDICTABLE "EASIER" QUESTIONS

PAPER 1:

Question 1 [Aim to score at least 18 marks for these equations and inequalities]
Lots of practice is needed for these straight forward questions.
Quadratic equations:

- easy factorising (can also use formula) (3)
- using formula (3)
- containing square root (4)

Simultaneous equations (5)
Exponential equation (5)
Quadratic inequality (3)

Number Pattern Questions [Aim to score at least 6 marks for these questions]
These questions usually include parts which only require substitution in the correct formula. Make sure you know which formula on the information sheet is used for which type of pattern and read questions carefully to know if a term or sum formula is required.

Functions Questions [Aim to score at least 6 marks from these graph-base questions]
These graph questions include at least a few basic questions such as calculating intercepts; stating the domain, range, asymptotes or axes of symmetry and substituting a point in an equation. These facts and methods are quick to learn and at least 6 marks can be so earned.

Finance Question [Aim to score at least 4 marks in this question as formulae are given]
Reading Finance questions carefully can earn marks.
Marks for interpreting interest rates and compounding periods (eg. 6% compounded monthly makes $i = \frac{0,06}{12}$ in the formulae) and for identifying the variables to be substituted in chosen formulae can be earned.

Basic Calculus Question [Aim to score at least 10 marks in this differentiation question]
Every paper includes a question requiring differentiation from first principles. Thorough practice of differentiating in this way can ensure 5 marks.
A further 7 marks is often available for quick differentiation which has 2 steps – writing the expression as a row of powers of x and then applying $D_x[x^n] = n x^{n-1}$.

Cubic Graph Question [Aim to score at least 5 marks in this question]
Calculating x-intercepts using $f(x) = 0$ or turning points using $f'(x) = 0$ or an inflection point using $f''(x) = 0$ is often asked and can be a source of these marks.

Probability Question [Aim to score at least 3 marks in this question]
Learn the definitions of "mutually exclusive", "complementary" and "independent" events as well as the counting principal. Note that the formula for P(A or B) and P(A) are on the information sheet. Knowing this information should be sufficient to gain at least 3 marks.

PAPER 2

Statistics Questions [Aim to score at least 15 marks in this question]
Statistics questions are easy and predictable. This section should be learnt thoroughly to take advantage of this.

Analytical Geometry Questions [Aim to earn at least 10 marks]
Formulae for the gradient, midpoint and length of a line segment as well as for the equation of a straight line or circle are provided on the information sheet. By identifying questions in which these formulae can be directly applied and substituting correctly, the necessary marks can be scored. Appropriate practice is vital during preparation.

Basic Trigonometry Questions [Aim to score at least 10 marks]
Every paper contains a question based on reading trigonometric ratios off a diagram. Ensure that the definitions of the ratios are fully known and practice this predictable question. The marks it provides are vital. Reducing all ratios to ratios of acute angles is another technique which is predictably tested as part of various questions. Being able to apply the A | S | T | C diagram can ensure that all of these marks are earned.
Compound and double angle identities are all provided on the information sheet. Applying these is often the first step even in more complex questions. Copying the appropriate identity from the information sheet for theses first steps can earn critical marks.

Trigonometric Graph Question [Aim to earn at least 3 marks]
Although parts of this question are often complex, there are also always basic questions regarding the period, amplitude, domain, range or asymptotes of a graph asked. Learning theses basics can earn critical marks.

Triangle Formulae Questions [Aim to earn at least 2 marks]
One of the triangles in this question is usually a right angled triangle. In working with it, all that is needed is applying the definitions of the trigonometric ratios in terms of opposite, adjacent and hypotenuse. Knowing theses definitions is necessary and can be the source of at least 2 marks.

Euclidean Geometry Questions [Aim to earn at least 12 marks]
A theorem is always asked for about 6 marks. There are only 7 to learn. Knowing all of them thoroughly is necessary and should guarantee at least 5 marks.
There is usually at least one easy question in which angles can be proved equal in one step by applying one theorem. Practice such questions and score at least 4 marks in this way.
There is a set way of presenting a similarity proof using angles. The question will often specify the triangles with their vertices listed in the correct order. This makes it possible to see which angles must be proved equal. Presenting the proof in the correct format can earn critical marks and should always be attempted.

REVISION FOR QUESTION 1

It is important to aim to score at least 18 marks for this predictable question.
Do the examples in the question column by applying the explanations in the method column.

Solve for x:

1.1	$x^2 + x - 6 = 0$	●Factorise (or substitute in the formula) if necessary ●Write down answer
1.2	$x^2 + x = 5$	●Write in standard form ●Copy the formula from the information sheet ●Substitute ●Write down the answer, rounded off to 2 decimal digits
1.3.1	$x^2 + x > 6$	●Write in standard form with the co-efficient of x^2 positive ●Factorise ●Write down answer. NOTE that as the sign is “ > ” , the answer is OUTSIDE the roots: $x < smaller\ root \quad OR \quad x > bigger\ root$
1.3.2	$-x^2 - x + 6 > 0$ $\therefore x^2 + x - 6 < 0$	●Write in standard form with the co-efficient of x^2 positive. NOTE that **the sign changes direction** as both sides are multiplied by a negative ●Factorise ●Write down answer. NOTE that as the sign is “ < ” , the answer is INSIDE the roots: $smaller\ root < x < bigger\ root$

Solve for x and y:

2	$x + 2y = 3 \quad and \quad x^2 - y^2 = 24$	●In the linear equation make one of the variables the subject. If possible, choose the one which avoids fractions. ●Substitute in the other equation ●Simplify and write in standard form ●Factorise ●Write down roots, here the values of y ●Substitute in linear equation to calculate values of x

An equation which includes a square root, a fraction or exponents is also often included. The NSC papers of 2024 included: $5^{2x} - 5^x = 0$; $2^{2x} - 2^{x+2} - 32 = 0$; $\sqrt{-2x+4} - x = 2$ and $\frac{x}{\sqrt{20-x}} = 1$.

For the purposes of this revision, consider the following exponential equation. Solve for *x*:

$5^x - 1 = \frac{2}{5^x}$ $\therefore 5^{2x} - 5^x = 2$	●Multiply through by the denominator to remove the fraction ●RECOGNISE as a quadratic equation as the index 2*x* is DOUBLE the index *x* ● Write equation in standard form. ●Factorise as a quadratic in 5^x ●The one root is $5^x = -1$ which is impossible as a positive can't equal a negative. The other root is $5^x . = 2$ use logs to calculate *x*.

and with a square root

$\sqrt{-2x+4} - 2 = x$	●write with square root term on its own on one side ●square both SIDES ●write in standard form ●factorise ●obtain *x*-values ●NB.: Test solutions and select only those which work

SOLUTIONS

1.1	$x^2+x-6=0$ $x^2+x-6=0$ $(x+3)(x-2)=0$ $x=-3\ or\ x=2$	●Factorise (or substitute in the formula) if necessary ●Write down answer
1.2	$x^2+x=5$ $\therefore x^2+x-5=0$ $x=\dfrac{-1\pm\sqrt{1^2-4(1)(-5)}}{2(1)}$ $\therefore x=-2{,}79\ or\ x=1{,}79$	●Write in standard form ●Copy the formula from the information sheet ●Substitute ●Write down the answer, rounded off to 2 decimal digits
1.3.1	$x^2+x>6$ $x^2+x-6>0$ $(x+3)(x-2)>0$ $x<-3\ or\ x>2$	●Write in standard form with the co-efficient of x^2 positive ●Factorise ●Write down answer. NOTE that as the sign is " > " , the answer is OUTSIDE the roots: $x<smaller\ root\quad OR\quad x>bigger\ root$
1.3.2	$-x^2-x+6>0$ $x^2+x-6<0$ $(x+3)(x-2)<0$ $-3<x<2$	●Write in standard form with the co-efficient of x^2 positive. NOTE that **the sign changes direction** as both sides are multiplied by a negative ●Factorise ●Write down answer. NOTE that as the sign is " < " , the answer is INSIDE the roots: $smaller\ root<x<bigger\ root$

2	$x+2y=3\quad and\quad x^2-y^2=24$ $x=3-2y$ $(3-2y)^2-y^2=24$ $\therefore 9-12y+4y^2-y^2=24$ $\therefore 3y^2-12y-15=0$ $\therefore y^2-4y-5=0$ $\therefore (y+1)(y-5)=0$ $\therefore y=-1\ or\ y=5$ $x=3-2(-1)=5$ or $x=3-2(5)=-7$	●In the linear equation make one of the variables the subject. If possible, choose the one which avoids fractions. ●Substitute in the other equation ●Simplify and write in standard form ●Factorise ●Write down roots, here the values of *y* ●Substitute in linear equation to calculate values of *x*

.

$5^{x} - 1 = \frac{2}{5^{x}}$ $\therefore 5^{2x} - 5^{x} = 2$ $\therefore 5^{2x} - 5^{x} - 2 = 0$ $\therefore (5^{x} + 1)(5^{x} - 2) = 0$ $\therefore 5^{x} \neq -1 \;\; but \;\; 5^{x} = 2$ $\therefore x = \log_{5} 2 = 0{,}43$	●Multiply through by the denominator to remove the fraction ●A quadratic equation with 5^{2x} as a term results. Write it in standard form. ●Factorise as a quadratic in 5^{x} ●The one root is $5^{x} = -1$ which is impossible as a positive can't equal a negative. The other root is $5^{x}. = 2$ use logs to calculate *x*.

$\sqrt{-2x+4} - 2 = x$ $\therefore \sqrt{-2x+4} = x + 2$ $\therefore -2x + 4 = x^{2} + 4x + 4$ $\therefore x^{2} + 6x = 0$ $\therefore x(x+6) = 0$ $\therefore x = 0 \;\; or \;\; x = -6$ TEST: $\sqrt{-2(0)+4} - 2 = 0 \quad \therefore x = 0$ $\sqrt{-2(-6)+4} - 2 = 2 \neq -6 \quad \therefore x \neq -6$	●write with square root term **on its own** on one side ●square both **SIDES,** not individual terms ●write in standard form ●factorise ●obtain *x*-values ●**NB**.: Test solutions and select only those which work

"EASY" QUESTION REVISION FOR NUMBER PATTERNS

Given the Geometric Series: 6 + 12 + 24 + . . .

Determine the general term, T_n	●Identify the TERM formula from the information sheet ●Identify the 1st term, *a* and common ratio, *r* (second term divided by first) ●Substitute in formula
Determine the sum of the first 10 terms	●Identify the SUM formula from the information sheet ●Substitute in formula with *n* as 10.

The general term of an Arithmetic Series is $T_n = 3n - 1$

Calculate the 20th term	●Substitute 20 for *n*
Calculate the sum of the first 20 terms	●Choose the correct SUM formula from the information sheet ●Identify *a* (substitute 1 in given term formula) and *d* (difference between second and first term) ●Substitute in SUM formula

14 ; 9 ; 6 ; 5 ; . . . is a Quadratic Pattern.
Show that $T_n = n^2 - 8n + 21$

	●Know that $T_n = an^2 + bn + c$ ●Calculate 2nd difference ●Show that half the second difference is 1 and therefore *a* is 1. ●the first term is 14, so formulate an equation by substituting 1 for *n* and equating it to 14 ●do the same for the second term by substituting 2 for *n* and equating to 9 ●Solve simultaneously for *b* and *c*.

SOLUTIONS

Given the Geometric Series: 6 + 12 + 24 + . . .

Determine the general term, T_n $T_n = ar^{n-1}$ $a = 6 \quad r = \frac{12}{6} = 2$ $T_n = 6.2^{n-1} \ OR \ 3.2^n$	●Identify the TERM formula from the information sheet ●Identify the 1st term, *a* and common ratio, *r* (second term divided by first) ●Substitute in formula
Determine the sum of the first 10 terms $S_n = \frac{a(r^n - 1)}{r - 1}$ $\therefore \ S_{10} = \frac{6\left(2^{10} - 1\right)}{2 - 1} = 6138$	●Identify the SUM formula from the information sheet ●Substitute in formula with *n* as 10.

The general term of an Arithmetic Series is $T_n = 3n - 1$

Calculate the 20th term $T_{20} = 3(20) - 1 = 59$	●Substitute 20 for *n*
Calculate the sum of the first 20 terms $T_1 = 3(1) - 1 = 2 \quad T_2 = 3(2) - 1 = 5$ $a = 2 \quad d = 3$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{20} = \frac{20}{2}[2(2) + (20-1).3] = 610$	●Choose the correct SUM formula from the information sheet ●Identify *a* (substitute 1 in given term formula) and *d* (difference between second and first term) ●Substitute in SUM formula

14 ; 9 ; 6 ; 5 ; . . . is a Quadratic Pattern.

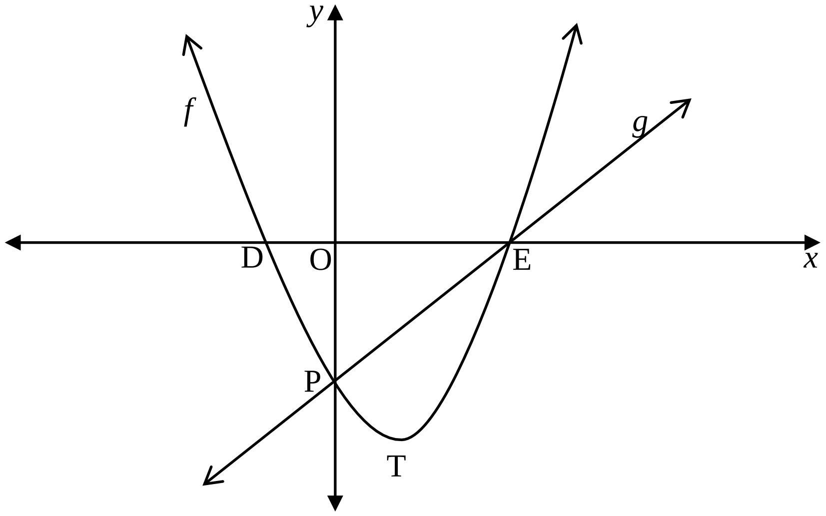
Show that $T_n = n^2 - 8n + 21$

14 9 6 5. $-5 \ -3 \ -1$ 2 2 second difference is 2 so *a* is half of 2 = 1 $1 + b(1) + c = 14 \quad \therefore b + c = 13 \ . \ . \ . (1)$ $2^2 + 2b + c = 9 \quad \therefore 2b + c = 5 \ . \ . \ . \ (2)$ (2) – (1): $b = -8$ Subst. in (1): $-8 + c = 13 \quad \therefore c = 21$	●Know that $T_n = an^2 + bn + c$ ●Calculate 2nd difference ●Show that half the second difference is 1 and therefore *a* is 1. ●the first term is 14, so formulate an equation by substituting 1 for *n* and equating it to 14 ●do the same for the second term by substituting 2 for *n* and equating to 9 ●Solve simultaneously for *b* and *c*.

“EASY” QUESTION REVISION : THE PARABOLA

The revision is based on Question 6 of the NSC May 2024 Paper 1.

The graph of $f(x) = x^2 - 2x - 3$ and $g(x) = mx + c$ are drawn below. D and E are the x-intercepts and P is the y-intercept of f. The turning point of f is T(1 ; −4). The graphs intersect at P and E.



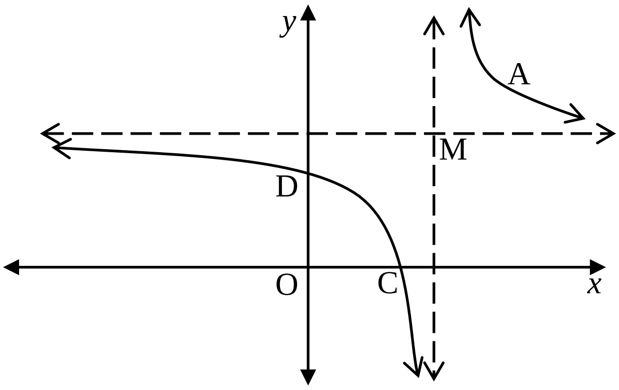
1. Write down the domain and range of f	●There is no restriction on the domain. ●The graph is concave up, so y is greater than or equal to the y-coordinate of the turning point.
2. Calculate the coordinates of P, D and E	●For y-intercept make $x = 0$. ●For x-intercepts make $f(x) = 0$ ●factorise ●identify D and E
3. Determine the equation of g	●Use $y = mx + c$ ●$m = \frac{y_E - y_P}{x_E - x_P}$ ●graphs have same y-intercept
4. $h(x) = x^2$ is given. $h(x) = f(x - p) + q$ Write down the values of p and q.	●For f to be translated to h, the turning point of f must be translated to (0 ; 0) ●Determine the horizontal and vertical movement required and deduce the values of p and q.
6. If the domain of h is restricted to $x \leq 0$, determine the equation of h^{-1}.	●Swop x and y. ●Make y the subject ●Choose the sign which results in h^{-1} having a range $y \leq 0$ [as h had negative domain]

SOLUTIONS

1. Write down the domain and range of f $domain$: $x \in R$ $range$: $y \geq -4$	●There is no restriction on the domain. ●The graph is concave up, so *y* is greater than or equal to the *y*-coordinate of the turning point.
2. Calculate the coordinates of P, D and E $y = 0^2 - 2(0) - 3 = -3$ $\therefore$ P(0 ; −3) $x^2 - 2x - 3 = 0$ $\therefore (x+1)(x-3) = 0$ $\therefore x = -1$ or $x = 3$ D(−1 ; 0) E(3 ; 0)	●For *y*-intercept make *x* = 0. ●For *x*-intercepts make $f(x)$ = 0 ●factorise ●identify D and E
3. Determine the equation of g P(0 ; −3) $m = 1$ $g(x) = x - 3$	●Use $y = mx + c$ ●$m = \frac{y_{\mathrm{E}} - y_{\mathrm{P}}}{x_{\mathrm{E}} - x_{\mathrm{P}}}$ ●graphs have same *y*-intercept
4. $h(x) = x^2$ is given. $h(x) = f(x - p) + q$ Write down the values of *p* and *q*. *graph must move 1 unit left and 4 units up* $h(x) = f(x + 1) + 4$ *so p=-1 and q=4*	●For f to be translated to *h*, the turning point of f must be translated to (0 ; 0) ●Determine the horizontal and vertical movement required and deduce the values of *p* and *q*.
6. If the domain of h is restricted to $x \leq 0$, determine the equation of h^{-1}. inverse: $x = y^2$ $\therefore y^2 = x$ $\therefore y = -\sqrt{x}$ $\therefore h^{-1}(x) = -\sqrt{x}$	●Swop *x* and *y*. ●Make *y* the subject ●Choose the sign which results in h^{-1} having a range $y \leq 0$ [as *h* had negative domain]

"EASY" QUESTION REVISION :: THE HYPERBOLA based on NSC May-June 2025 paper

The graph of $f(x)=\dfrac{4}{x-3}+4$ is drawn below. M is the point where the asymptotes intersect.
D is the y-intercept, C is the x-intercept and A is the point on f which is closest to M.

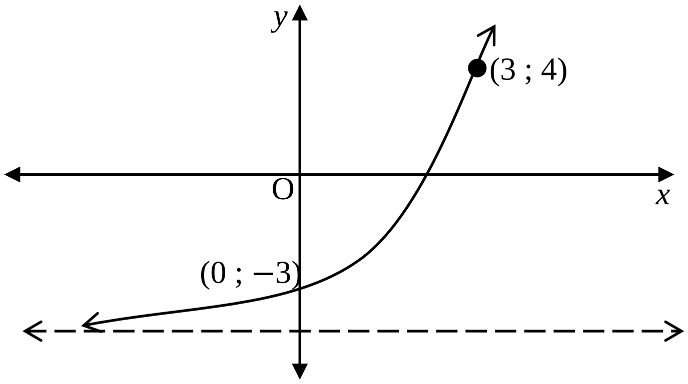


1	Write down the equations of the asymptotes	●Note that these must be EQUATIONS with the form $x = a\,number$ and $y = a\,number$
2	Write down the coordinates of M.	●M is the point where the asymptotes intersect and also the point where the axes of symmetry intersect
3	Determine the equations of the axes of symmetry	●They have the form $y = x + c$ and $y = -x + k$ As they pass through M, substitute M to calculate c and k. Method 2: Translate $y = \pm x$ in the same way the hyperbola is translated to get $y = (x - 3) + 4$ and $y = -(x - 3) + 4$
4	Calculate the coordinates of C and D	●for C, make $y = 0$ ●for D. make $x = 0$
5	State the domain and range of f.	●This can be learnt, only the asymptotes must be excluded.

SOLUTIONS

1	Write down the equations of the asymptotes $x = 3$ $y = 4$	●Note that these must be EQUATIONS with the form $x = a\,number$ and $y = a\,number$
2	Write down the coordinates of M. (3 ; 4)	●M is the point where the asymptotes intersect and also the point where the axes of symmetry intersect
3	Determine the equations of the axes of symmetry $4 = 3 + c \quad \therefore c = 1 \quad \therefore y = x + 1$ $4 = -3 + k \quad \therefore k = 7 \quad \therefore y = -x + 7$ Method 2: $y = (x - 3) + 4 \quad \therefore y = x + 1$ $y = -(x - 3) + 4 \quad \therefore y = -x + 7$	●They have the form $y = x + c$ and $y = -x + k$ As they pass through M, substitute M to calculate c and k. Method 2: Translate $y = \pm x$ in the same way the hyperbola is translated to get $y = (x - 3) + 4$ and $y = -(x - 3) + 4$
4	Calculate the coordinates of C and D For C, $y = 0$: $0 = \frac{4}{x-3} + 4 \;\therefore 0 = 4 + 4x - 12 \therefore 4x = 8 \;\therefore x = 2$ C(2 ; 0) For D, $x = 0$: $y = \frac{4}{0-3} + 4 = \frac{8}{3} = 2{,}67$ D(0 ; 2,67)	●for C, make $y = 0$ ●for D. make $x = 0$
5	State the domain and range of f. Domain: $x \neq 3\,; x \in R$ Range: $y \neq 4\,; y \in R$	●This can be learnt, only the asymptotes must be excluded.

"EASY" QUESTION REVISION: EXPONENTIAL AND LOG CURVES based in part on NSC May-June 2025



The diagram shows the graph of $f(x) = p^x + q$. Two points on the graph are (0 ; −3) and (3 ; 4).

1	Calculate the values of p and q	●Substitute (0 ; −3) to calculate q. ●Then (3 ; 4) to calculate p
2	Write down the equation of the asymptote	●equation is $y = q$
3	Write down the domain and range of f.	●This should be learnt
4	Explain why $f'(x) > 0$ for $x \in R$	●consider the gradient of the graph
5	Calculate the x-intercept	●Solve $2^x - 4 = 0$
5	The graph of f is translated to become $h(x) = 2^x$. Determine the equation of h^{-1}.	●For inverse: replace x with y and y with x. ●make y the subject
6	Write down the equation of the line about which h and h^{-1} are symmetrical	●This is always $y = x$

SOLUTIONS

1	Calculate the values of p and q Subst. (0 ; −3): $-3 = p^0 + q \quad \therefore -3 = 1 + q \quad \therefore q = -4$ Subst. (3 ; 4): $4 = p^3 - 4 \quad \therefore \; 8 = p^3 \quad \therefore p = 2$	●Substitute (0 ; −3) to calculate q. ●Then (3 ; 4) to calculate p
2	Write down the equation of the asymptote $y = -4$	●equation is $y = q$
3	Write down the domain and range of f . *domain is* $x \in R$ *range is* $y > -4 \; ; y \in R$	●This should be learnt
4	Explain why $f'(x) > 0$ for $x \in R$ *the graph is increasing and so has positive gradient for* $x \in R$	●consider the gradient of the graph
5	Calculate the x-intercept $2^x - 4 = 0$ $\therefore \; 2^x = 4$ $\therefore 2^x = 2^2$ $\therefore x = 2$ intercept is (2 ; 0)	●Solve $2^x - 4 = 0$
5	The graph of f is translated to become $h(x) = 2^x$. Determine the equation of h^{-1}. $x = 2^y$ $\therefore y = \log_2 x$ $h^{-1}(x) = \log_2 x$.	●For inverse: replace x with y and y with x. ●make y the subject
6	Write down the equation of the line about which h and h^{-1} are symmetrical $y = x$	●This is always $y = x$

“EASY” QUESTION REVISION: FINANCE adapted from NSC May-June 2023

1. A company bought a photocopier for R150 000 on 1 July 2022. They will use the old photocopier as a trade-in when they replace it with a similar new photocopier in 5 years' time on 30 June 2027.

The average rate of inflation over the 5 years will be 6,5% p.a. Determine the price of a similar new photocopier in 5 years' time	●Recognise that $A = P(1+i)^n$ is the formula to be used ●Substitute
Calculate the trade-in value of the old photocopier after 5 years, if it depreciates at a rate of 9% p.a. on a straight-line method	●Read carefully that depreciation is *straight line* method and choose $A = P(1-ni)$ as the formula. Then substitute.
The company sets up a sinking fund to cover the R123 013 required to buy the new photocopier. The fund earns interest at the rate of 7,85% p.a., compounded monthly. The company made its first monthly deposit on 31 July 2022 and will continue to do so until 30 June 2027. How much should be deposited at the end of each month?	●Note that the formula to use with a sinking fund is the Future Value formula and select it from the information sheet ●Note that interest is compounded monthly so $i = \frac{0{,}0785}{12}$ ●Note that deposits are made at the end of the month and that there are 60 ●Substitute and use the calculator to find x

2.

Andrew borrowed R200 000 from a bank. The bank charges interest at 9,5% p.a., compounded quarterly. Andrew will make repayments of R10 000 at the end of every 3 months. How long will it take Andrew to settle the loan?	●Note that with loans the Present Value formula is used and select it from the information sheet. ●Note that compounding is quarterly, so $i = \frac{0{,}095}{4}$ ●Note that payments are at the end of each quarter and that $x = 10\ 000$. ●Calculating n is a bit more complex and requires logs, but earlier marks are easy to earn

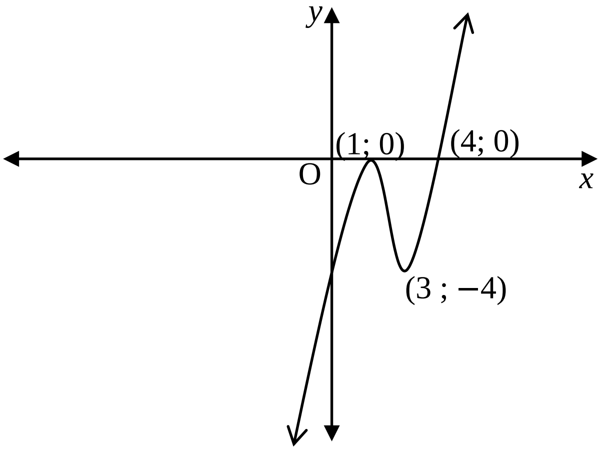
SOLUTIONS

The average rate of inflation over the 5 years will be 6,5% p.a. Determine the price of a similar new photocopier in 5 years' time $A = 150\ 000\ (1+0{,}065)^{5} = R205\ 513$	●Recognise that $A = P(1+i)^{n}$ is the formula to be used ●Substitute
Calculate the trade-in value of the old photocopier after 5 years, if it depreciates at a rate of 9% p.a. on a straight-line method $A = 150\ 000\ (1 - 5 \times 0{,}09) = R\ 82\ 500$	●Read carefully that depreciation is straight line method and choose $A = P(1-ni)$ as the formula. Then substitute.
The company sets up a sinking fund to cover the R123 013 required to buy the new photocopier. The fund earns interest at the rate of 7,85% p.a., compounded monthly. The company made its first monthly deposit on 31 July 2022 and will continue to do so until 30 June 2027. How much should be deposited at the end of each month? $F = \frac{x\left[(1+i)^{n} - 1\right]}{i}$ $\therefore 123\,013 = \frac{x\left[\left(1+\frac{0{,}0785}{12}\right)^{60} - 1\right]}{\frac{0{,}0785}{12}}$ $\therefore x = R\,1\,680{,}73$	●Note that the formula to use with a sinking fund is the Future Value formula and select it from the information sheet ●Note that interest is compounded monthly so $i = \frac{0{,}0785}{12}$ ●Note that deposits are made at the end of the month and that there are 60 ●Substitute and use the calculator to find x

Andrew borrowed R200 000 from a bank. The bank charges interest at 9,5% p.a., compounded quarterly. Andrew will make repayments of R10 000 at the end of every 3 months. How long will it take Andrew to settle the loan? $P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$ $\therefore 200\ 000 = \frac{10\ 000\left[1 - \left(1+\frac{0{,}095}{4}\right)^{-n}\right]}{\frac{0{,}095}{4}}$ $\therefore \left(1+\frac{0{,}095}{4}\right)^{-n} = 0{,}525$ $\therefore -n = \log_{\left(1+\frac{0{,}095}{4}\right)} 0{,}525 = -27{,}45$ $\therefore n = 28\ \mathit{terms} = 7\ \mathit{years}$	●Note that with loans the Present Value formula is used and select it from the information sheet. ●Note that compounding is quarterly, so $i = \frac{0{,}095}{4}$ ●Note that payments are at the end of each quarter and that x = 10 000. ●Calculating n is a bit more complex and requires logs, but earlier marks are easy to earn

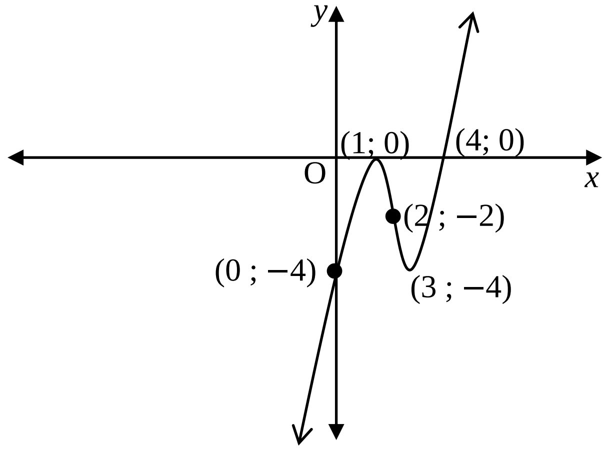
“EASY” QUESTION REVISION for CALCULUS based in part on NSC May 2025 paper.

1	Determine $f'(x)$ from FIRST PRINCIPLES if $f(x)=x^2-2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{\qquad\qquad}{h}$ $=\lim_{h\to 0}\frac{\qquad\qquad}{h}$ $=\lim_{h\to 0}\frac{\qquad\qquad}{h}$ $=\lim_{h\to 0}\frac{\qquad\qquad}{h}$ $=\lim_{h\to 0}(\qquad\qquad)$ $=$	●Get definition from information sheet ●Make sure correct notation is maintained and remember **brackets** when substituting ●multiply out brackets ●simplify numerator ●extract *h* as common factor ●divide ●take limit
2	Determine: $\frac{dy}{dx}$ if $y=3x^2-\frac{4}{x}$ $Ans.:y=$ $\therefore\ \frac{dy}{dx}=$ $D_x\left[-2\sqrt{x}\left(x^2+1\right)\right]$ $=D_x[\qquad\qquad]$ $=$	●Use correct notation ●First write *y* as row of powers of *x*; then differentiate NOTE: $\frac{4}{x^n}=4x^{-n}$ $\sqrt[n]{x^m}=x^{\frac{m}{n}}$ ●first write as row of powers of *x* inside [ ] by writing $\sqrt{x}$ as $x^{\frac{1}{2}}$ and multiplying bracket ●write down derivatives as answers
3	Given that $f'(x)=-3x^2+4$ Write down the gradient of: f at $x=5$ f' at $x=5$	●the gradient of *f* is given by f'. So determine $f'(5)$ ●the gradient of f' is given by f''

4	$f(x) = x^3 - 6x^2 + 9x - 4$ Calculate the coordinates of the turning points Determine the coordinates of the *x*-intercepts	●put $f'(x) = 0$ and solve ●Notice that $f(1) = 0$, making $(x - 1)$ a factor of $f(x)$ ●Factorise and solve $f(x) = 0$
	Indicate the coordinates of the inflection point and *y*-intercept.  	●For inflection point $f''(x) = 0$ OR use midpoint of line segment joining turning points. ●For *y*-intercept, make *x* = 0.
	Determine the values of *x*, for which the graph is concave up.	●Method 1: Look at graph, locate inflection point and see on what side of it graph has concave up shape. ●Method 2: Use $f''(x) > 0$ and calculate.

SOLUTIONS

1	Determine $f'(x)$ from FIRST PRINCIPLES if $f(x) = x^2 - 2$ $f'(x) = \lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $= \lim\limits_{h\to 0} \frac{(x+h)^2-2-[x^2-2]}{h}$ $= \lim\limits_{h\to 0} \frac{x^2+2xh+h^2-2-x^2+2}{h}$ $= \lim\limits_{h\to 0} \frac{2xh+h^2}{h}$ $= \lim\limits_{h\to 0} \frac{h(2x+h)}{h}$ $= \lim\limits_{h\to 0} (2x+h)$ $= 2x$	●Get definition from information sheet ●Make sure correct notation is maintained and remember **brackets** when substituting ●multiply out brackets ●simplify numerator ●extract *h* as common factor ●divide ●take limit
2	Determine: $\frac{dy}{dx}$ if $y = 3x^2 - \frac{4}{x}$ $Ans.: y = 3x^2 - 4x^{-1}$ $\therefore \frac{dy}{dx} = 6x + 4x^{-2}$ $D_x\left[-2\sqrt{x}\left(x^2+1\right)\right]$ $= D_x\left[-2x^{\frac{5}{2}} - 2x^{\frac{1}{2}}\right]$ $= -5x^{\frac{3}{2}} - x^{-\frac{1}{2}}$	●Use correct notation ●First write *y* as row of powers of *x*; then differentiate ●first write as row of powers of *x* inside [ ] ●write down derivatives as answers
3	Given that $f'(x) = -3x^2 + 4$ Write down the gradient of: f at $x = 5$: $m = f'(5) = -3(5)^2 + 4 = -71$ f' at x = 5: $f''(x) = -6x$ gradient = $f''(5) = -6(5) = -30$	●the gradient of *f* is given by f'. So determine $f'(5)$ ●the gradient of f' is given by f''
4	$f(x) = x^3 - 6x^2 + 9x - 4$ Calculate the coordinates of the turning points $f'(x) = 0$ $\therefore 3x^2 - 12x + 9 = 0$ $\therefore x^2 - 4x + 3 = 0$ $\therefore (x-1)(x-3) = 0$ $\therefore x = 1 \quad or \quad x = 3$ $\therefore y = 1^3 - 6(1)^2 + 9(1) - 4 \; or \; y = 3^3 - 6(3)^2 + 9(3) - 4$ $= 0$ $= -4$ (1 ; 0) (3 ; −4)	●put $f'(x) = 0$ and solve

	Determine the coordinates of the *x*-intercepts $f(1)=0 \quad \therefore (x-1)$ is a factor $x^3-6x^2+9x-4=0$ $(x-1)(x^2-5x+4)=0$ $\therefore (x-1)(x-1)(x-4)=0$ $\therefore x=1 \;\; or \;\; x=4$ (1 ; 0) (4 ; 0)	●Notice that $f(1)=0$, making $(x-1)$ a factor of $f(x)$ ●Factorise and solve $f(x)=0$
	Indicate the coordinates of the inflection point and *y*-intercept. 	●For inflection point $f''(x)=0$ OR use midpoint of line segment joining turning points. ●For *y*-intercept, make *x* = 0.
	Determine the values of *x*, for which the graph is concave up. $x>2$ $f''(x)>0$ $\therefore 6x-12>0$ $\therefore 6x>12$ $\therefore x>2$	●Method 1: Look at graph, locate inflection point and see on what side of it graph has concave up shape. ●Method 2: Use $f''(x)>0$ and calculate.

FUNDAMENTAL COUNTING PRINCIPAL

codes must indicate whether **repetitions** are allowed or not

Example: 4-letter codes from A,B,C,D

reps allowed: $4 \times 4 \times 4 \times 4 = 4^4$

reps not allowed: $4 \times 3 \times 2 \times 1 = 4!$

letter arrangements must indicate whether same letters are **distinguishable or not**

Example: arrangements of MINIMUM

repeats distinguishable: 7!

not distinguishable: $\frac{\overset{\text{all letters}}{7!}}{\underbrace{3!}_{\text{repeated M's}}\times\underbrace{2!}_{\text{repeated I's}}}$

selecting people can't have repetitions

special conditions grouping certain individuals together create **entities**

Example: 4 white, 3 silver and 2 green cars are to be parked in a row. The silver cars must park next to one another.
3 silver form an **entity**. The entity plus 6 other cars = 7 items to be arranged making: 7! ways .
Multiply this by the number of arrangements within entity: 3!
So answer: 7! × 3! ways are possible

conditions can restrict options for certain positions

Example: Seating 5 people if Joe must sit in 1st seat and Donald in last seat

$$\underbrace{1}_{\text{Joe}} \times \underbrace{3 \times 2 \times 1}_{\text{rest}} \times \underbrace{1}_{\text{Donald}}$$

requiring that individuals NOT be next to one another, requires **gaps**

Example: 4 red and 3 silver cars must be parked so that no silver cars are next to one another. There are 5 **gaps:** _R_R_R_R_ for the first silver, 4 for the second and 3 for the third to choose from. The red can be arranged in 4! ways.
Answer:

$$5 \times 4 \times 3 \times 4! = 1440 \; ways$$

USING DEFINITIONS AND RULES

P(A OR B) = P(A) + P(B) – P(A and B)

A AND B ARE MUTUALLY EXCLUSIVE implies P(A and B) = 0

A AND B ARE INDEPENDENT implies P(A and B) = P(A) × P(B)

A AND B ARE COMPLEMENTARY IF P(A) + P(B) = 1   B can be written as A', the complement of A

A IS CERTAIN implies P(A) = 1

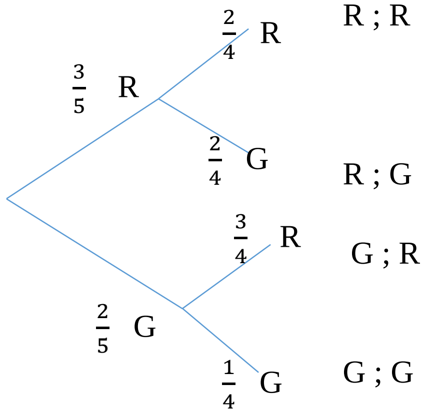
IF P(B) = x then P(not B) = $1 - x$ P(not B) can be written P(B')

P(only B) can be calculated as P(B) – P(A and B) or as P(A or B) – P(A)

TOOLS

TREE DIAGRAM

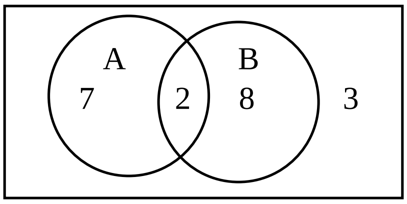
Example 3 red and 2 green balls in bag. One is drawn and not put back, then another is drawn



Probability that 2 balls of same colour drawn:

$$\frac{3}{5}\times\frac{2}{4}+\frac{2}{5}\times\frac{1}{4}=\frac{8}{20}=0,4$$

VENN DIAGRAM



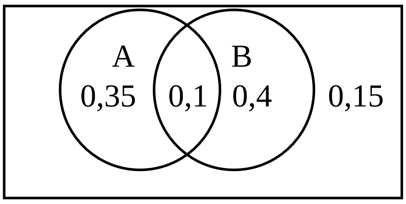
P(A or B) = $\frac{17}{20}$

P(A only) = $\frac{7}{20}$

P(not A) = $\frac{11}{20}$

P(B and not A) = $\frac{8}{20}$

diagram can indicate probabilities



CONTINGENCY TABLE

	PHYS SCIENCE	GEOGRAPHY	CAT	TOTAL
MATHS	78	10	2	90
MATH LIT	0	42	18	60
TOTAL	78	52	20	150

P(MATHS) = $\frac{90}{150}$ P(GEOGRAPHY) = $\frac{52}{150}$ P(MATHS AND GEOGRAPHY) = $\frac{10}{150}$

P(M) × P(G) = 0,21

$\neq$ 0,07 P(M AND G)

So taking Mathematics and taking Geography are not independent

PRACTICE EXERCISE

USING COUNTING PRINCIPLE

There are 6 seats in a row.

How many ways are there for 6 people to sit in them?	●The first person has 6 options, the 2nd has 5, the 3rd has 4 and so on. Apply the counting principle.
If Adam must sit in the first seat and Eve in the 6th seat, in how many ways?	●There is one option for the 1st seat and 1 for the last. This leaves 4 options for the 2nd , 3 for the 3rd, 2 for the 4th and 1 for the 5th . Apply the counting principle.

USING DEFINITIONS AND RULES
Given that P(A) = 0,4 P(B) = x P(A or B) = 0,7

Calculate the value of x in each of the following cases:

1 A and B are mutually exclusive	●Being mutually exclusive means P(A and B) = 0 ●Get the formula for P(A or B) from the information sheet ●Substitute and calculate x
2 A and B are independent	● A and B are independent means P(A and B) = P(A) X P(B) ●Proceed as in 1.

SOLUTIONS

How many ways are there for 6 people to sit in them? $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6! = 720$	●The first person has 6 options, the 2nd has 5, the 3rd has 4 and so on. Apply the counting principle.
If Adam must sit in the first seat and Eve in the 6th seat, in how many ways? $1 \times 1 \times 4 \times 3 \times 2 \times 1 = 24$	●There is one option for the 1st seat and 1 for the last. This leaves 4 options for the 2nd , 3 for the 3rd, 2 for the 4th and 1 for the 5th . Apply the counting principle.

1 A and B are mutually exclusive P(A or B) = P(A) + P(B) – P(A and B) 0,7 = 0,4 + x – 0 $\therefore x = 0{,}3$	●Being mutually exclusive means P(A and B) = 0 ●Get the formula for P(A or B) from the information sheet ●Substitute and calculate x
2 A and B are independent P(A or B) = P(A) + P(B) – P(A and B) 0,7 = 0,4 + x – $0{,}4 \times x$ 0,3 = 0,6x $\therefore x$ = 0,5	● A and B are independent means P(A and B) = P(A) X P(B) ●Proceed as in 1.

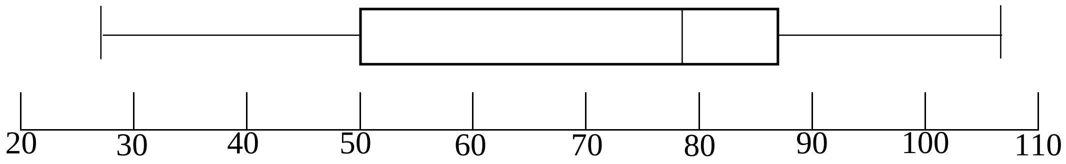
“EASY” QUESTION REVISION FOR STATISTICS

UNGROUPED DATA.

The table below shows the monthly rainfall (in mm.) of a coastal city for 12 months.
The data is arranged in ascending order.

27	28	44	56	68	77
81	85	87	87	108	108

This data is depicted in the box and whisker diagram drawn below.



Calculate: 1. the mean monthly rainfall 2. the standard deviation of the data	●Set your calculator mode on STAT choose 1-VAR and enter the data into the table. Push AC, then SHIFT 1 and choose 4 VAR then 2 $\bar{x}$ yields the mean. AC, SHIFT 1; 4 VAR; then 3 σ yields the standard deviation
Determine the number of months in which rainfall was within 1 standard deviation of the mean	●Mean measures the average and standard deviation the spread. Determine the interval $(\bar{x} - \sigma\ ;\ \bar{x} + \sigma)$ and count how many data points lie within it.
Calculate the median, 5-number summary, the range and the inter-quartile range of the data	●the median indicates the average, while the range and inter-quartile range indicate spread. ●For the median arrange the data in ascending order and then find the middle. Here it is midway between the 6th and 7th point: $\frac{77+81}{2}$ ●The range is: the maximum – the minimum ●Q_1 is the middle of the points below the median. Here midway between the 3rd and 4th point $\frac{44+56}{2}$. Q_3 is the middle of the points above the median. Here midway between the 9th and 10th point. ●the IQR = $Q_3 - Q_1$
Describe the skewness of the data. Give a reason for your answer.	●mean > median or longer spread on right of median implies **skewed right** mean < median or longer spread on left of median implies **skewed left** **symmetric** spread if both sides the same

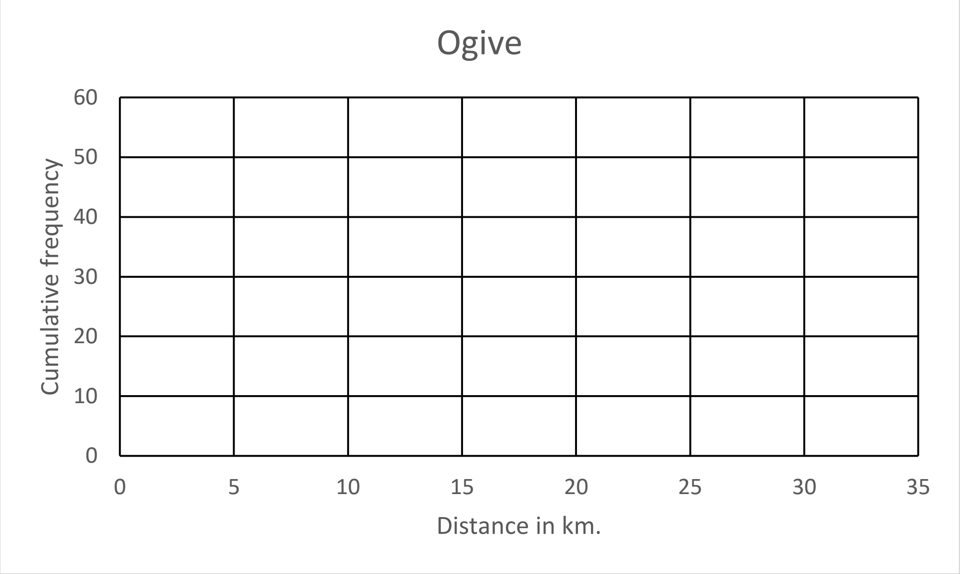
SOLUTIONS

Calculate: 1. the mean monthly rainfall 2. the standard deviation of the data $\bar{x} = 71{,}33 \quad \sigma = 26{,}34$	●Set your calculator mode on STAT choose 1-VAR and enter the data into the table. Push AC, then SHIFT 1 and choose 4 VAR then 2 $\bar{x}$ yields the mean. AC, SHIFT 1; 4 VAR; then 3 σ yields the standard deviation
Determine the number of months in which rainfall was within 1 standard deviation of the mean. rainfall in interval (44,99 ; 97,67) for 7 months	●Mean measures the average and standard deviation the spread. Determine the interval $(\bar{x} - \sigma\,;\ \bar{x} + \sigma)$ and count how many data points lie within it.
Calculate the median, 5-number summary, the range and the inter-quartile range of the data median = $\frac{77+81}{2} = 79$ range = 108 – 27 = 81 $Q_1 = \frac{44+56}{2} = 50 \qquad Q_3 = 87$ IQR = 87 – 50 =37 **5-number summary**: min.; Q_1; median; Q_3; max. 27; 50; 79; 87; 108	●the median indicates the average, while the range and inter-quartile range indicate spread. ●For the median arrange the data in ascending order and then find the middle. Here it is midway between the 6th and 7th point: $\frac{77+81}{2}$ ●The range is: the maximum – the minimum ●Q_1 is the middle of the points below the median. Here midway between the 3rd and 4th point $\frac{44+56}{2}$. Q_3 is the middle of the points above the median. Here midway between the 9th and 10th point. ●the IQR = $Q_3 - Q_1$
Describe the skewness of the data. Give a reason for your answer. skewed left as mean < median OR from diagram, gap between minimum and median is longer, so data more spread out on left.	●mean > median or longer spread on right of median implies skewed right mean < median or longer spread on left of median implies skewed left

GROUPED DATA
The question is based on the May 2024 NSC Paper 2.

Fifty athletes need to access suitable training facilities. The table below shows the distances in km., that they need to travel to obtain access to suitable training facilities.

DISTANCE (x km)	**NUMBER OF ATHLETES [FREQUENCY]**	**CUMULATIVE FREQUENCY**
$0 \leq x < 5$	3	3
$5 \leq x < 10$	7	
$10 \leq x < 15$	20	
$15 \leq x < 20$	12	
$20 \leq x < 25$	5	
$25 \leq x < 30$		50



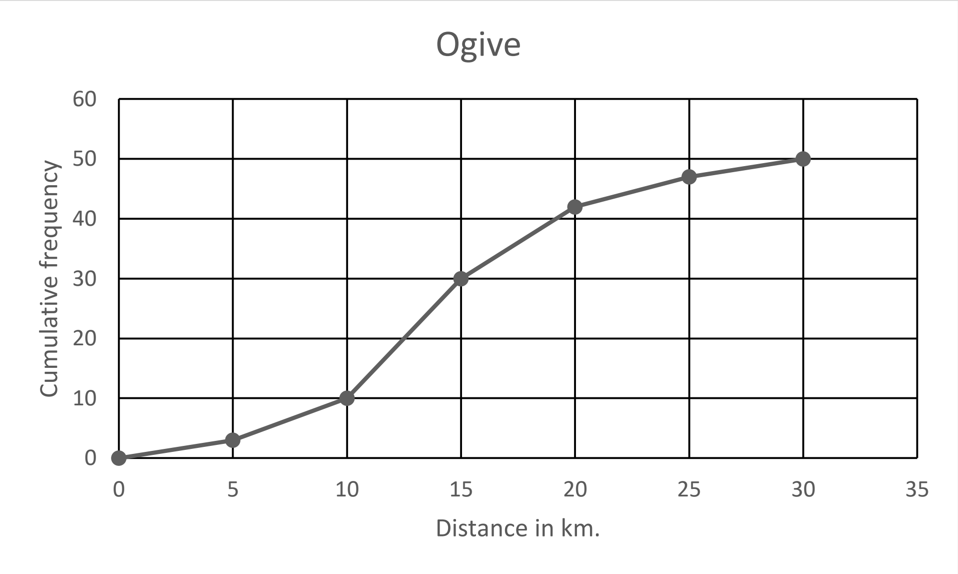
1. Complete the table	●For the 2nd row cumulative frequency add 7 to 3 and so on
2. On the grid provided, draw a cumulative frequency curve (ogive) to represent the above data	●Plot the UPPER limit of interval vs the cumulative frequency. Join points with curve (not lines) ●Ground curve at lower limit of 1st interval
3. State the modal class	●Check which class has the highest frequency.
4. Calculate the median and IQR of the above data.	●The middle of 50 is midway between 25 and 26. To find median: go across from 25,5 on the y-axis till the curve is met. The corresponding x-value is the median. ●The middle of the first 25 athletes is the 13th . Go across from 13 on the y-axis till the curve is met. The corresponding x-value is Q_1. The middle of the second 25 athletes is the 38th. Go across from 38 on the y-axis till the curve is met. The corresponding x-value is Q_3. IQR = $Q_3 - Q_1$

5. How many athletes need to travel more than 8 km.? What percentage is this?	●Go up from 8 on the *x*-axis till the curve is met. The corresponding *y*-value is then subtracted from 50. ●For percentage take the answer, divide by the total of 50 and multiply by 100 to obtain %.

6. Calculate the estimated mean.	●Multiply the *midpoint* of each interval by the corresponding *frequency*. Add these products and divide by 50.

SOLUTIONS

DISTANCE (*x* km)	**NUMBER OF ATHLETES [FREQUENCY]**	**CUMULATIVE FREQUENCY**
$0 \leq x < 5$	3	3
$5 \leq x < 10$	7	10
$10 \leq x < 15$	20	30
$15 \leq x < 20$	12	42
$20 \leq x < 25$	5	47
$25 \leq x < 30$	3	50



3. State the modal class $10 \leq x < 15$	●Check which class has the highest frequency.
4. Calculate the median and IQR of the above data. median = 13 (accept 14) IQR = 18 – 11 = 7 [allow error margin of 1 for each quartile]	●The middle of 50 is midway between 25 and 26. To find median: go across from 25,5 on the y-axis till the curve is met. The corresponding *x*-value is the median. ●The middle of the first 25 athletes is the 13th . Go across from 13 on the y-axis till the curve is met. The corresponding *x*-value is Q_1. The middle of the second 25 athletes is the 38th. Go across from 38 on the *y*-axis till the curve is met. The corresponding *x*-value is Q_3. IQR = $Q_3 - Q_1$

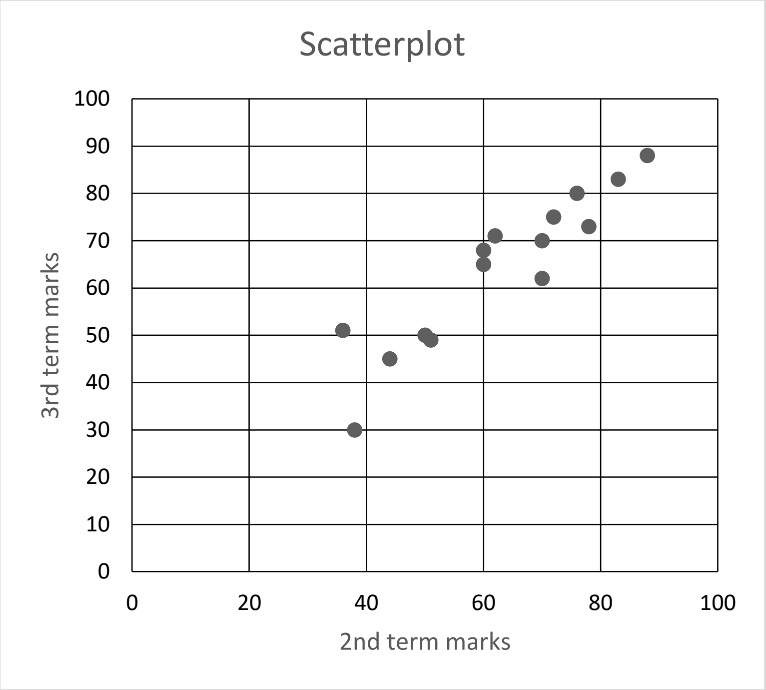
5. How many athletes need to travel more than 8 km.? read off: 7 are less, [accept 8 o r 9] so 50 – 7 = 43 [accept 42 or 41] 43 / 50 × 100 = 86%	●Go up from 8 on the *x*-axis till the curve is met. The corresponding *y*-value is then subtracted from 50. ●For percentage take the answer, divide by the total of 50 and multiply by 100 to obtain %.

6. Calculate the estimated mean. $\bar{x} = \frac{3 \times 2{,}5 + 7 \times 7{,}5 + 20 \times 12{,}5 + 12 \times 17{,}5 + 5 \times 22{,}5 + 3 \times 27.5}{50}$ = 14,3 km.	●Multiply the *midpoint* of each interval by the corresponding *frequency*. Add these products and divide by 50.

REGRESSION

A teacher wishes to establish the relationship between the 2nd and 3rd term marks of a class.
The marks are shown in the table and plotted on the scatterplot.

2nd term	**3rd term**
60	68
60	65
36	51
70	70
62	71
72	75
88	88
70	62
78	73
51	49
38	30
76	80
83	83
44	45
50	50



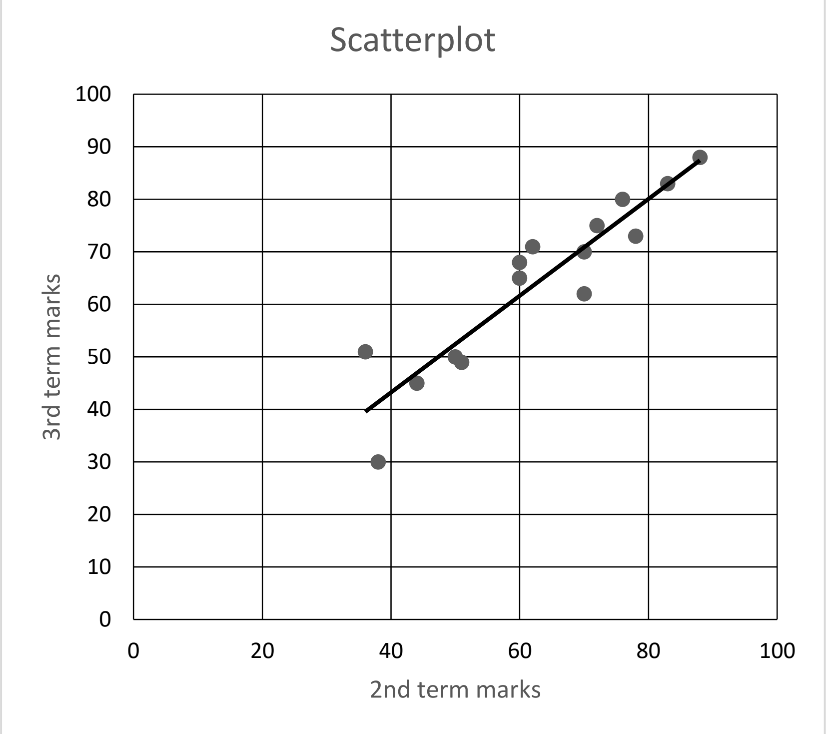
Determine the equation of the least squares regression line for the data	● On the calculator push mode, then choose stat, then A + BX. Enter the values from the data sets in the X and Y columns. Push AC; then shift STAT; then choose Reg; then choose A or B as needed for the equation $\hat{y} = \mathrm{A} + \mathrm{B}x$
Write down the correlation coefficient of the data	●Proceed as above, but choose *r* on the final screen.

Predict the 3rd term mark of a learner who scores 40 in the 2nd term	●Substitute 40 for *x* in the equation of the least squares regression line.
Why can the above prediction be regarded as being reliable?	●There are 2 possible reasons: 40 must be within the domain of the second term data and the value of *r* must be reasonably close to 1 to indicate a strong correlation
Draw the least squares regression line on the above scatterplot	●Calculate 2 points using its equation, plot and join the points. ●Make sure the line starts opposite the first point on the scatter plot and ends opposite the last

Describe the association between the 2nd and 3rd term marks.	●Base this on the value of *r* Mention whether it is positive or negative and very strong, strong, moderate or weak.
The equation of the least squares regression line is used to predict the 3rd term mark of a learner with a 2nd term mark of 20. Why would this prediction **not** be reliable?	●The prediction is unreliable if the mark is outside the domain of the data set or if there is a weak correlation

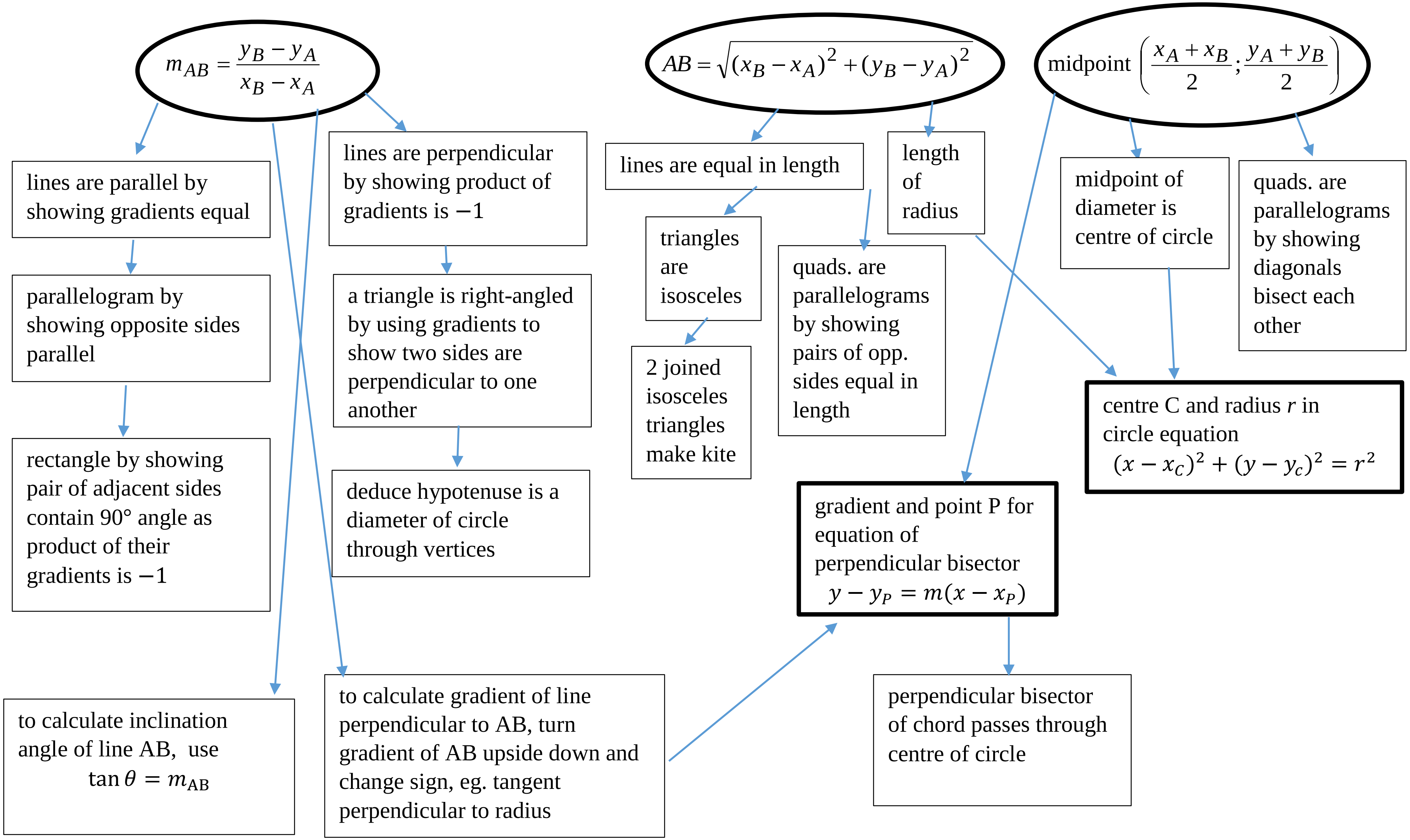
SOLUTIONS

Determine the equation of the least squares regression line for the data A = 6,44 B = 0,92 $\hat{y} = 6,44 + 0,92x$	● On the calculator push mode, then choose stat, then A + BX. Enter the values from the data sets in the X and Y columns. Push AC; then shift STAT; then choose Reg; then choose A or B as needed for the equation $\hat{y} = \mathrm{A} + \mathrm{B}x$
Write down the correlation coefficient of the data *r* = 0,92	●Proceed as above, but choose *r* on the final screen.
Predict the 3rd term mark of a learner who scores 40 in the 2nd term 6,44 + 0,92 (40) = 43	●Substitute 40 for *x* in the equation of the least squares regression line.
Why can the above prediction be regarded as being reliable? *40 is in data domain and correlation is very strong as r = 0,92*	●There are 2 possible reasons: 40 must be within the domain of the second term data and the value of *r* must be reasonably close to 1 to indicate a strong correlation



Describe the association between the 2nd and 3rd term marks. *very strong positive*	●Base this on the value of r Mention whether it is positive or negative and very strong, strong, moderate or weak.
The equation of the least squares regression line is used to predict the 3rd term mark of a learner with a 2nd term mark of 20. Why would this prediction **not** be reliable? *20 is outside domain of least squares regression line / extrapolation unreliable*	●The prediction is unreliable if the mark is outside the domain of the data set or if there is a weak correlation

REVISION ANALYTICAL GEOMETRY



$$m_{AB} = \frac{y_B - y_A}{x_B - x_A}$$

lines are parallel by showing gradients equal

parallelogram by showing opposite sides parallel

rectangle by showing pair of adjacent sides contain 90° angle as product of their gradients is -1

lines are perpendicular by showing product of gradients is -1

a triangle is right-angled by using gradients to show two sides are perpendicular to one another

deduce hypotenuse is a diameter of circle through vertices

to calculate inclination angle of line AB, use $\tan\theta = m_{\text{AB}}$

to calculate gradient of line perpendicular to AB, turn gradient of AB upside down and change sign, eg. tangent perpendicular to radius

$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

lines are equal in length

triangles are isosceles

2 joined isosceles triangles make kite

quads. are parallelograms by showing pairs of opp. sides equal in length

length of radius

midpoint $\left(\frac{x_A + x_B}{2} ; \frac{y_A + y_B}{2}\right)$

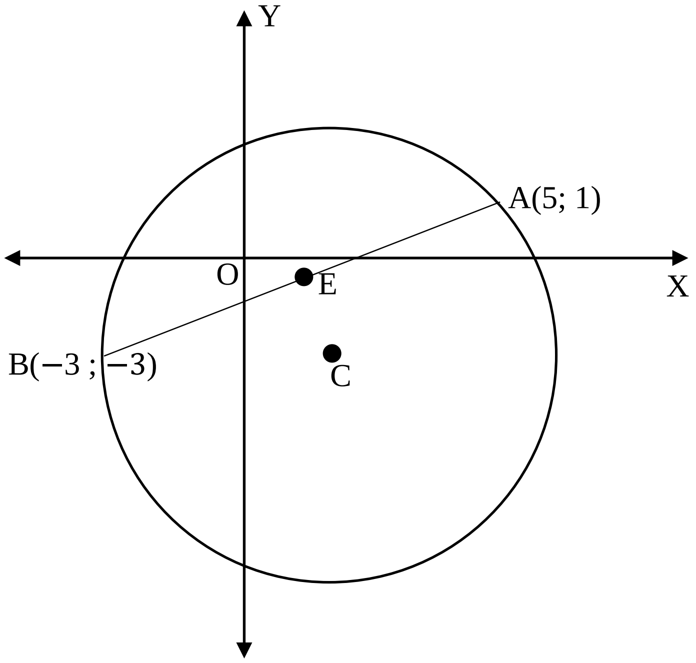
midpoint of diameter is centre of circle

quads. are parallelograms by showing diagonals bisect each other

centre C and radius r in circle equation $(x - x_c)^2 + (y - y_c)^2 = r^2$

gradient and point P for equation of perpendicular bisector $y - y_P = m(x - x_P)$

perpendicular bisector of chord passes through centre of circle



C is the centre of circle $(x-2)^2+(y+3)^2=25$.
B(−3 ; −3) and A(5 ; 1) are points on the circle.

1. Write down the coordinates of C.
2. Calculate E if it is the midpoint of AB
3. Determine the equation of the tangent at A
4. Calculate the length of EA
5. Determine the equation of a circle with E as centre and EA as radius.

SOLUTIONS

1 *Read off the equation*

2. *Get formula from information sheet and substitute*

3. *The formula for a straight line equation is on the information sheet*

●*Determine the gradient of radius CA using the gradient formula from the information sheet*

●*The tangent is perpendicular so turn the gradient of CA upside down and change its sign. This is m in the formula for the straight line.*

●*The point to substitute in the formula is A.*

4. *Use the distance formula from the information sheet*.

5. *The formula for a circle is on the information sheet. The point to use for the centre is E and use length EA for r*

SOLUTIONS

1 (2 ; −3)

2. $\left(\frac{-3+5}{2}\ ;\ \frac{-3+1}{2}\right)\ so\,E\,(1;-1)$

3. *The formula for a straight line equation is on the information sheet*

$$y-y_1=m(x-x_1)$$

●*Determine the gradient of radius CA using the gradient formula from the information sheet*

$$m=\frac{1-(-3)}{5-2}=\frac{4}{3}$$

●*The tangent is perpendicular so turn the gradient of CA upside down and change its sign. This is m in the formula for the straight line.*

$$m=-\frac{3}{4}$$

●*The point to substitute in the formula is* A(5 ; 1)

$$y-1=-\frac{3}{4}(x-5)$$

$$\therefore\ y=-\frac{3}{4}x+\frac{19}{4}$$

4. *Use the distance formula from the information sheet.*

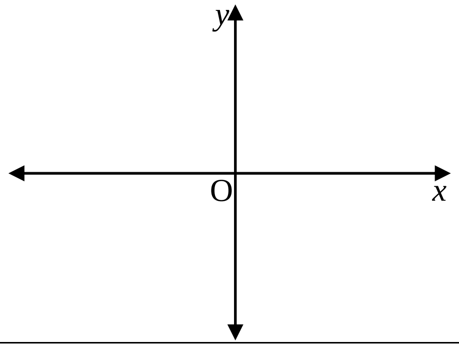
EA = $\sqrt{(5-1)^2+(1-(-1))^2}=\sqrt{20}$

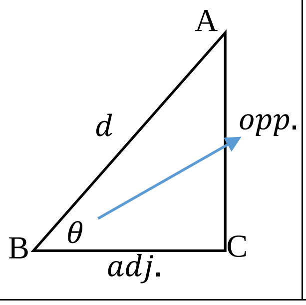
5. *The formula for a circle is on the information sheet. The point to use for the centre is E and use length EA for r*

$$(x-1)^2+(y+1)^2=(\sqrt{20})^2$$

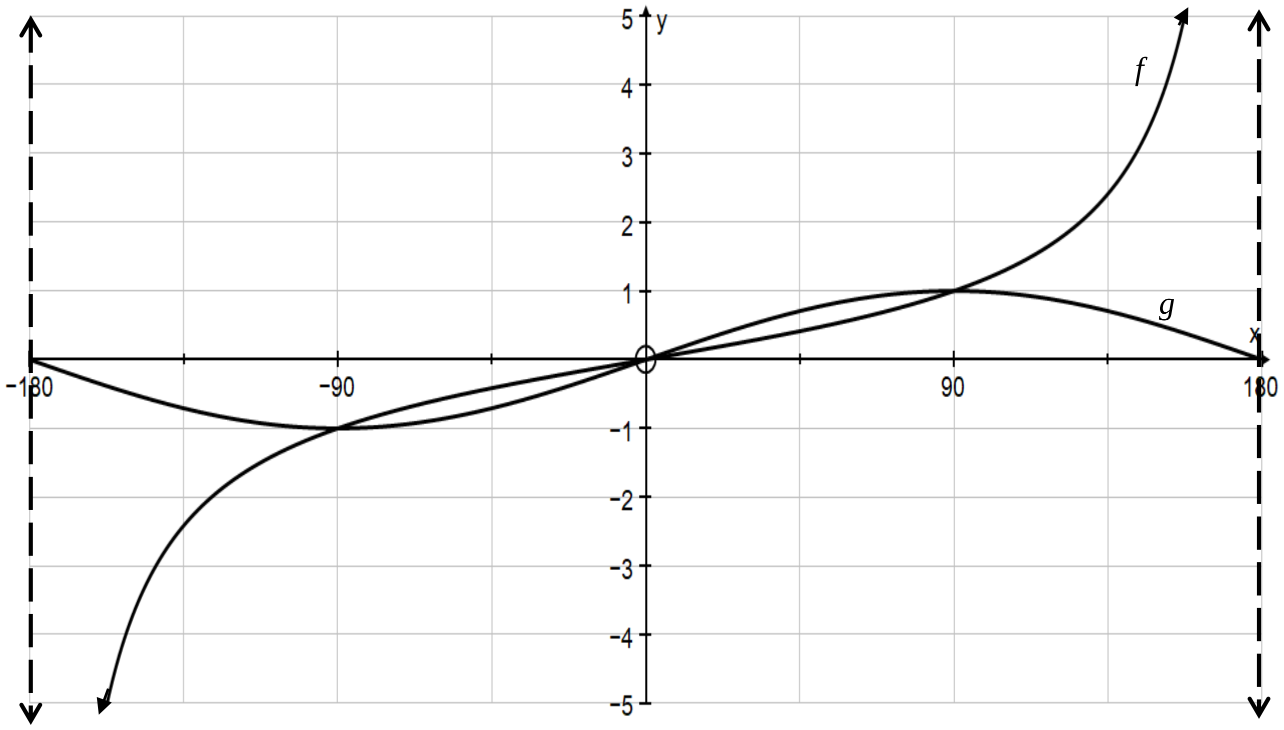
$$\therefore\ (x-1)^2+(y+1)^2=20$$

“EASY” QUESTION REVISION FOR TRIGONOMETRY

	Every angle has two arms which meet at a point called the vertex. In trigonometry the initial arm is placed on the positive x-axis and the vertex at the origin. The terminal arm rotates in an anti-clockwise direction to form positive angles. (and in a clockwise direction for negative angles) *For a given trig ratio, the quadrant of the terminal arm of the angle is determined by whether the ratio is positive or negative.*	
	Given that sin A = $\frac{4}{5}$ and A > 90°. Depict the information on a diagram.  	●**A** is the **angle** and it is given as greater than 90°**.** As the *ratio, sin A*, is positive, the terminal arm of the angle must be in the 2nd quadrant. ●Use the fact that the definition of sine of an angle is $\frac{y-coordinate}{r}$ to indicate the value of *r* and the *y*-coordinate of a point on the terminal arm. Note that the *signs* of the coordinates must match the quadrant. In the 2nd quadrant *x* is negative and *y* is positive ●Calculate *x* using the fact that $r^2 = x^2 + y^2$
	Use the diagram to determine the values of: 1. tan A and cos A 2. tan (360° – A) 3. cos 2A 4. sin (A + 45°)	The values of *x*, *y* and *r* can be read off the diagram. ●Use the definitions tan A = $\frac{y-coordinate}{x-coordinate}$ and cos A = $\frac{x-coordinate}{r}$ ●Use reduction formulae which are summarised in $\begin{array}{c\|c} S & A \\ \hline T & C \end{array}$ ●Use appropriate double angle formula (given on information sheet) ●Use appropriate compound angle formula (given on information sheet)

	*In a **right-angled** triangle the trigonometric functions are defined in terms of the sides.* sin A = $\frac{opposite}{hypotenuse}$ cos A = $\frac{adjacent}{hypotenuse}$ tan A = $\frac{opposite}{adjacent}$ **NOTE: A is an ANGLE while sin A, cos A and tan A are RATIOS**	
	Refer to the diagram and express in terms of θ and *d* 1. AC 2. BC  	●Check which of the ratios involve the required side and *d*, then use it ●Make the required side the subject

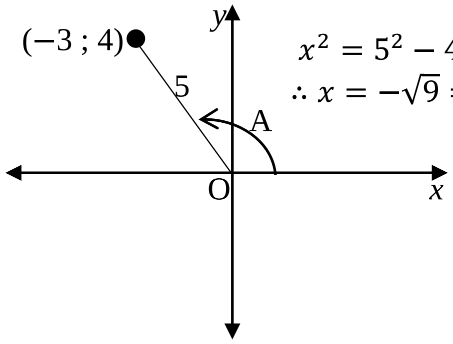
	If cos 20° = p, express in terms of p , without using a calculator 1. tan 20°	●Draw a right angled triangle with p the side adjacent to the 20° angle and 1 the hypotenuse. Calculate the opposite side using Pythagoras ●Read tan 20° off using the opposite over adjacent definition
	2. cos 200° + sin 340°	●Apply the reduction formulae, then read off answers from the triangle

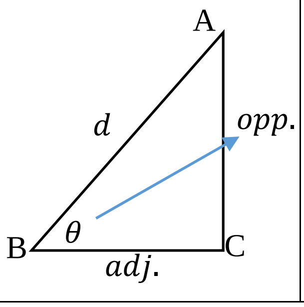


The diagram shows the graphs of $f(x)=\tan\left(\frac{1}{2}x\right)$ and $g(x)=\cos(x-d)$

Write down the period of f and the period of g.	●Look at the diagram and see how many degrees it takes for one copy (cycle) of each graph.
Write down the range of g.	●Look at the diagram and see the extent of the y-values.
Write down the equations of the asymptotes of f	●Read off the graph. Note equations have the form $x = . . .$
For which values of x is $g(x) > 0$?	●Look where the graph is above the x-axis.

SOLUTIONS

	Given that sin A = $\frac{4}{5}$ and A > 90°. Depict the information on a diagram.  $x^2 = 5^2 - 4^2 = 9$ $\therefore x = -\sqrt{9} = -3$	●**A** is the **angle** and it is given as greater than 90°**.** As the *ratio, sin A*, is positive, the terminal arm of the angle must be in the 2nd quadrant. ●Use the fact that the definition of sine of an angle is $\frac{y-coordinate}{r}$ to indicate the value of *r* and the *y*-coordinate of a point on the terminal arm. Note that the *signs* of the coordinates must match the quadrant. In the 2nd quadrant *x* is negative and *y* is positive ●Calculate *x* using the fact that $r^2 = x^2 + y^2$
	Use the diagram to determine the values of: 1 tan A and cos A tan A = $\frac{4}{-3}$ cos A = $\frac{-3}{5}$ 2 tan (360° − A) = −tan A [tan in 4th quad. is negative] $= -\left(\frac{4}{-3}\right) = \frac{4}{3}$ 3 cos 2A = $\cos^2 A - \sin^2 A$ $= \left(\frac{-3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{9-16}{25} = \frac{-7}{25}$ 4 sin (A + 45°) = sin A cos 45° + cos A sin 45° $= \frac{4}{5}\times\frac{1}{\sqrt{2}} + \frac{-3}{5}\times\frac{1}{\sqrt{2}} = \frac{1}{5\sqrt{2}}$	The values of *x*, *y* and *r* can be read off the diagram. ●Use the definitions tan A = $\frac{y-coordinate}{x-coordinate}$ and cos A = $\frac{x-coordinate}{r}$ ●Use reduction formulae which are summarised in $\begin{array}{cc} S & A \\ \hline T & C \end{array}$ ●Use appropriate double angle formula (given on information sheet) ●Use appropriate compound angle formula (given on information sheet)

	Refer to the diagram and express in terms of θ and *d* 1 AC AC is *opp* and *d* is *hyp* $\sin\theta = \frac{AC}{d}$ $\therefore$ AC = $d\sin\theta$ 2 BC (the *adjacent*) $\cos\theta = \frac{BC}{d}$ $\therefore$ BC = $d\cos\theta$  	●Check which of the ratios involve the required side and *d*, then use it ●Make the required side the subject

<table>
<tr>
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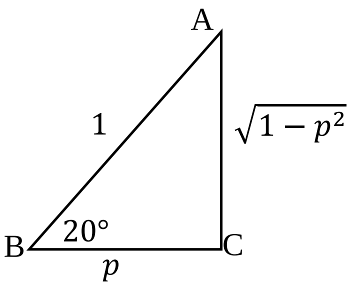
<td>
If cos 20° = p, express in terms of p , without using a calculator

1. tan 20°

$= \frac{\sqrt{1-p^2}}{p}$



2. cos 200° + sin 340°

$= -\cos 20° - \sin 20°$

$= -p - \sqrt{1-p^2}$
</td>
<td>
●Draw a right angled triangle with p the side adjacent to the 20° angle and 1 the hypotenuse. Calculate the opposite side using Pythagoras

●Read tan 20° off using the opposite over adjacent definition

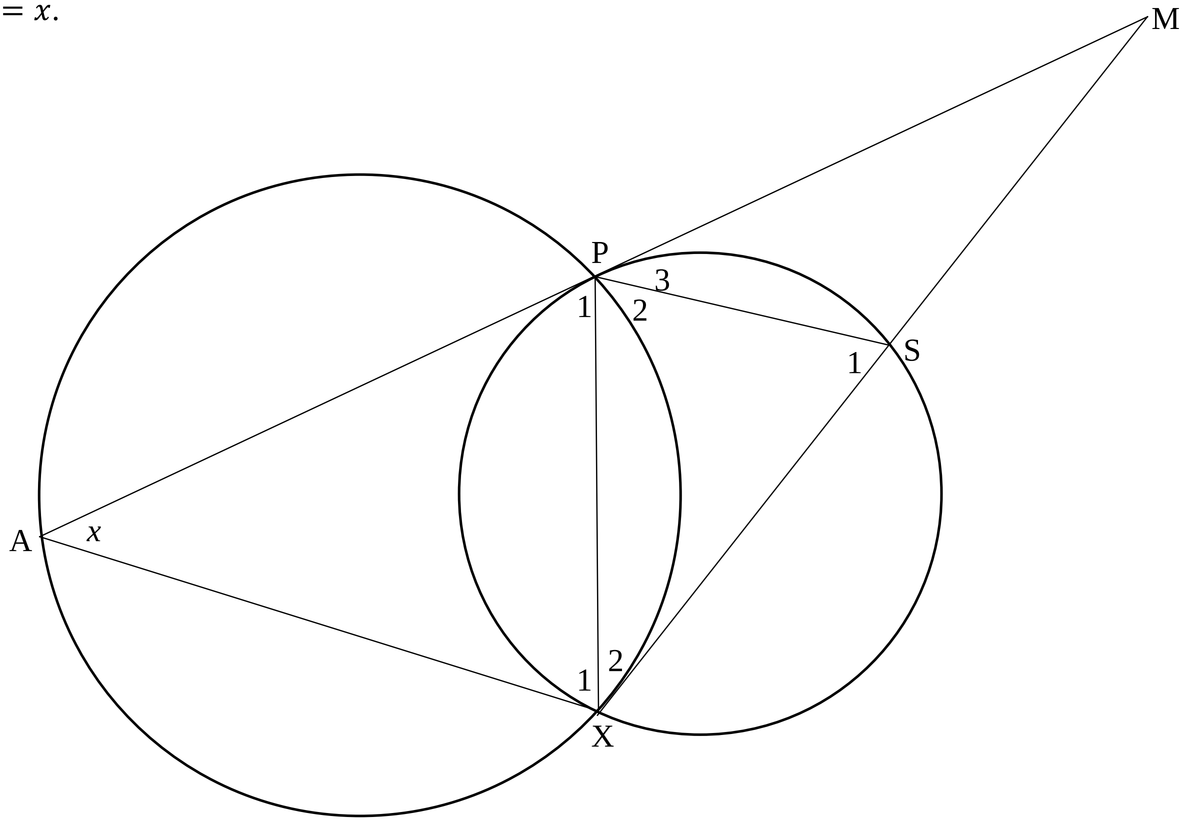
●Apply the reduction formulae, then read off answers from the triangle
</td>
</tr>
</table>

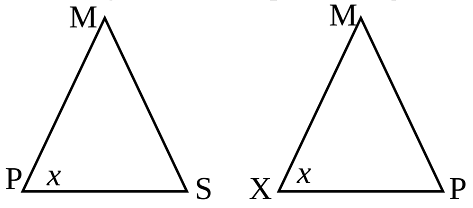
Write down the period of f and 360° can also be calculated as $\frac{180°}{\frac{1}{2}}$ the period of g. 360°	●Look at the diagram and see how many degrees it takes for one copy (cycle) of each graph.
Write down the range of g. $-1 \le y \le 1$	●Look at the diagram and see the extent of the y-values.
Write down the equations of the asymptotes of f $x = -180°$ $x = 180°$	●Read off the graph. Note equations have the form $x = . . .$
For which values of x is $g(x) > 0$? $0° < x < 180°$	●Look where the graph is above the x-axis.

“EASY” QUESTION REVISION: EUCLIDEAN GEOMETRY

●MAKE SURE THE THEOREM PROOFS ARE LEARNT. They are provided in a further document. The following example shows how “easy” marks are accessible in a question.

In the diagram, line MX is **tangent** to circle PAX at X, and **PS | | AX**.
PX, AP and AX are chords of the larger circle.
$\hat{A} = x$.



Write down, with reasons, two other angles equal to *x*.	●*There are two angles which in one step can be identified as x. Such one step questions are accessible. The parallel lines make corresponding angles equal, while the tangent and chord also create an angle equal to* $\hat{A}$.
Prove, giving reasons, that ΔMPS | | | ΔMXP. In ΔMPS and ΔMXP 1. 2. ∴ ΔMPS | | | ΔMXP (equiangular)	●*Drawing rough sketches of the triangles with vertices named in the correct order will show which angles must be proven equal*  A pair of angles have already been identified as *x* There is also a pair of common angles. ●Set out the proof as indicated
Prove that $MP^2 = MS . MX$	●*Even this slightly more complex question follows directly from the sketches by equating the ratio of the left sides to that of the right sides. and then cross multiplying. This makes the marks accessible.*

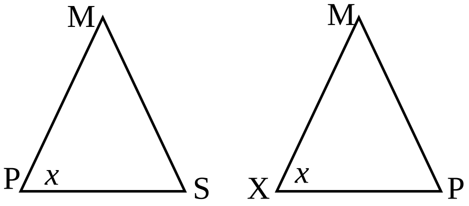
SOLUTIONS

<table>
<tr><td>Write down, with reasons, two other angles equal to *x*.

$\hat{P}_3 = x$ (corresp. <s ; PS | | AX)
$\hat{X}_2 = x$ (tangent ; chord)</td><td>●There are two angles which in one step can be identified as *x*. Such one step questions are accessible. The parallel lines make corresponding angles equal, while the tangent and chord also create an angle equal to $\hat{A}$.</td></tr>
<tr><td>Prove, giving reasons, that ΔMPS | | | ΔMXP.

In ΔMPS and ΔMXP
1. $\hat{M}$ is common

2. $M\hat{P}S = M\hat{X}P$ (proven both equal *x*)

∴ ΔMPS | | | ΔMXP (equiangular)</td><td>●Drawing rough sketches of the triangles with vertices named in the correct order will show which angles must be proven equal

A pair of angles have already been identified as *x*
There is also a pair of common angles.
●Set out the proof as indicated</td></tr>
<tr><td>Prove that $MP^2 = MS . MX$

$\frac{MP}{MX} = \frac{MS}{MP}$ (ΔMPS | | | ΔMXP)

∴ $MP^2 = MS . MX$</td><td>●Even this slightly more complex question follows directly from the sketches by equating the ratio of the left sides to that of the right sides and then cross multiplying. This makes the marks accessible.</td></tr>
</table>